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1. ይህን መትማሪ መልዕክት የተማሪ መጽሐፍ በጥንቃቄ ያዙ ለማስታወቂያ ከወጣ ስትሆን ያዙ ያለል።
2. ይህ መጆን እንዳይበላሽ እህቶች የሚካት ንብረት ከማስታወቂያ ያለል። መጽሐፉ እንዳይበላሽ ወይም እንዳይጠፋ በጣም መጠንቀቅ አለባችሁ። መጽሐፉን ለመያዝ ከዚህ በታች የተዘረዘሩትን መመሪያዎች ተግባራዊ አድርጉ፣
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5. መጽሐፉን ለመጠቀም ስትዘጋጁ በቅድሚያ እርጥበት የሌለውን ንጹህ መሆኑን አረጋግጡ፤
6. በመጽሐፉ ውስጥ ይህ ሽፋን ወይም ገጾች እንዳይጠሉ መጽሐፉን በምስክር ጋድም አድርጋችሁ አስቀምጡት፤ ከዚያም ጥቁት ገጾችን ቀስ እያላችሁ በየተራ ግለጡ፤ በግራና በቀኝ እ JACK የያዛችሁትን የመጽሐፉን ክፍል በኃይል በኃይል እንዲ የሚወሰ እንዲ የሚችሉ።

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 Maharawi
Mathematics

GRADE 7

Student Textbook

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The Scholars Council
Rationales for the Curriculum Reform

Curriculum relevance and its appropriateness to develop higher order thinking skills have been a subject of discussion in Ethiopia for many years. Various studies have been conducted to identify and propose reform ideas to the general education quality and efficiency problems. The main and most comprehensive ones are the Education Roadmap study and the Cambridge Education study. These studies indicated that the general education curriculum is staffed with many subjects; some textbooks are overloaded with factual content; school contents are not adequately related with students’ lives; and indigenous knowledge and real world problems are not integrated in the school curriculum. The studies also showed that the curriculum does not integrate ICT and has gaps to meet the needs of children with special educational needs. In addition, the studies demonstrated that curriculum materials do not thoroughly cultivate 21st Century skills and competencies such as lifelong learning, critical thinking, problem solving, creative and innovative thinking, communication and cooperative skills, leadership and decision making skills, technological skills, cultural identity and international citizenship. Consequently, these studies recommended the need to reform the curriculum.

Based on the recommendations, extended discussions and consultative meetings were conducted at national and regional levels with relevant stakeholders, teachers, parents, and educational leaders. Following these consultations, a new national curriculum framework, content flow chart, learning competencies, and syllabus have been prepared national level. Based on such documents, student textbooks and teacher guides of different subjects are prepared. The mathematics textbooks intend to engage students in the formulation and construction of mathematics knowledge and skill based on their day to day experiences and previous knowledge. Learners are expected to actively take part in drawing their prior knowledge and experience in the learning process.
How to Use the Book

The book is prepared to enable student learn in active and participatory manner using their prior learning and experiences from the immediate environment. Students and teachers carry out activities and solve problems using their experiences and knowledge. In this process, students not only learn mathematical concepts and ideas but also develop the necessary learning to learn skills. Such practice also helps students to deeply understand the contents. To this end, teachers are expected to teach using the proposed learning and teaching strategies and learning processes. It is essential that students and teachers appreciate the processes involved in learning the contents and not merely focus on memorization of concepts and mathematical procedures. Hence, teachers are expected to employ the proposed techniques and implement all activities as they are designed by considering the objectives and contents of the textbook. In addition, teacher can select and use other methods and approaches based on students’ capacity and needs.

Dear Students!

You have to use the textbook with care. Learning largely determines the future of a generation. Learning is a base for any social, human, and economic development. The textbook contents and activities are designed to promote your active participation in class. By carrying out and studying all the activities, contents and questions provided in the textbook, you are required to develop deep understandings and skills. Effort, exercise, and perseverance are important to succeed in your academic career. You will enjoy and find learning mathematics to be fun! Make sure that you bring your textbook to class and use it during the teaching and learning process.

Dear Parents!

Textbooks have significant roles to facilitate student learning. Thus, you are required to help and advice students to handle and use textbooks with care. Moreover, you are expected to motivate students to take textbooks to school and direct and support them to work the activities given by teachers. You should also visit your child’s school to discuss with teachers about learning and behavioral change, identify gaps, and correct them through follow up and advising.
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Learning Outcomes:

After completing this unit, you will be able to:

- Describe elements of a set using listing and verbal methods.
- Determine if a given set is empty, finite or infinite.
- Determine if two or more sets are equal by examining their elements.
- List all subsets for a given set using proper notation.
- Describe the union and intersection of two sets.
- Compare the union and intersection of two given sets using set notation and Venn diagram.

Key Terms

- set, element
- listing method
- verbal method
- empty set
- finite set
- infinite set
- equal sets
- subset
- union
- intersection
- Venn diagram
INTRODUCTION

In your surroundings there are different groups or collections of objects or individuals. For example group of all Grade 7 students in your school is a collection of individuals; group of all teachers in your school is another collection of individuals. In this unit you will learn about some sets and properties of sets.

1.1 Sets and Elements of Sets

From your responses in Activity 1.1, observe that, things are grouped together with a certain property in common such as family members, a collection of clothes, and fingers of a hand. All these groups are well defined.

Activity 1.1

Consider the following groups or collections of objects.

Group F: mother, father, son, daughter
Group C: jacket, hat, t-shirt, shoes, trousers
Group H: thumb, index, middle, ring, little

1. Describe the nature of each group.
2. How many members does each group contain?
3. What are the members of each group? Can you list some more members of the groups?

From your responses in Activity 1.1, observe that, things are grouped together with a certain property in common such as family members, a collection of clothes, and fingers of a hand. All these groups are well defined.

Definition:

A set is a well-defined collection of objects or individuals. The objects in the set are called its elements or members of the set.

By well-defined, we mean that, given a collection and an object or individual, we have to say that the object or the individual is in the collection or not, without any ambiguity.
Example 1.1

Identify each of the following collections as a set or not a set.

a. The collection of natural numbers less than 10.

b. The group of tall students in a certain school.

c. The collection of vowel letters in the English alphabet.

d. A group of rich people in Bahir Dar.

Solution:

a. The collection of natural numbers less than 10 is a well-defined collection. The natural numbers 1, 2, 3, 4, 5, 6, 7, 8 and 9 are in the collection and anything other than these nine numbers is not in the collection.

b. The group of tall students in a certain school is not well-defined. It is because, tallness is relative, someone may be tall for some group, but not for others.

c. The vowels in the English alphabet are a, e, i, o, and u are well-defined and anything other than these five letters is not in the collection.

d. The group of rich people in Bahir Dar is not well-defined because richness is relative.

Note:

1. Sets are denoted by capital letters like A, B, C, etc, and elements of a set are denoted by small letters like a, b, c, x, y, z, etc.

2. Given a set A and an object x,

   i. If x is an element of A, then we denote this relation by “\( x \in A \)” and read as “x is an element of A” or “x belongs to A”.

   ii. If x is not an element of A, then we denote this relation by “\( x \notin A \)” and read as “x is not an element of A” or “x does not belong to A”.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>( x \in A )</td>
<td>is an element of</td>
</tr>
<tr>
<td>( x \notin A )</td>
<td>is not an element of</td>
</tr>
</tbody>
</table>

3. Set notation uses braces or curly brackets, \{\}, with elements separated by commas. So the set of clothes to wear would be listed as: \{jacket, hat, t-shirt, shoes, trousers\}. 
Example 1.2

If A is the set of natural numbers less than 6, then A can be written as A = \{1, 2, 3, 4, 5\}. In this case, 1 \in A, 2 \in A, 3 \in A, 4 \in A and 5 \in A, but 7 \notin A and also 10 \notin A.

*Every object in a set is unique: The same object cannot be included in the set more than once.*

For example, if A = \{1, 2, 3, 4\}, then every element of A is included not more than once. We do not write A = \{1, 2, 3, 4, 4\} or A = \{1, 1, 2, 3, 4\} or A = \{1, 2, 2, 3, 4, 4\}.

Example 1.3

What is the set of all fingers of a hand?

**Solution:**

Fingers of a hand are thumb, index, middle, ring and little.

Thus, the set of fingers of a hand is given as: \{thumb, index, middle, ring, little\}.

Example 1.4

Find the set of all the primary colors.

**Solution:**

The primary colors are the three basic colors; red, blue, yellow, that can be combined to make different colors.

Therefore, the set of primary colors is \{red, blue, yellow\}.

Example 1.5

Determine the set D as the set of all days in a week.

**Solution:**

There are seven days in a week: Monday, Tuesday, Wednesday, Thursday, Friday, Saturday and Sunday.

Therefore, D = \{Monday, Tuesday, Wednesday, Thursday, Friday, Saturday, Sunday\}.

Exercise 1.1

1. Which of the following collections are sets? Justify your answer.
   a. The set of all beautiful birds in Ethiopia.
   b. The set of all intelligent students in a certain school.
UNIT 1: INTRODUCTION TO SETS

c. The set of natural numbers less than 10000.
d. The set of all prime factors of 2000.
e. The set of Grade 8 students in 2015 in Ethiopia.

2. Identify each of the following statements as true or false.

a. \( 1 \in \{-1, 0, 1, 2\} \)
b. \( a \in \{a, b, c, d, e\} \)
c. \( 3 \notin \{\text{factors of 48}\} \)

1.2 Ways to Describe Sets

Activity 1.2

Describe each of the following sets.

a. \( O \) is the set of odd numbers less than 12.
b. \( S \) is the set of Grade 7 students in Ethiopia.

From your responses in Activity 1.2, observe that, in set \( O \), you can list all the elements of the given set. On the other hand, in \( S \), it is difficult to list the elements of the set.

Listing or Roster Method

Listing or roster notation is a list of elements, separated by commas, enclosed in curly braces. The curly braces are used to indicate that the elements written between them belong to that set.

Example 1.6

Let \( V \) be the set of all vowels in the English alphabet. Write \( V \) using listing method.

**Solution:**

The vowel letters in the English alphabet are \( a, e, i, o \) and \( u \). 

So the set \( V \) using listing method is given by \( V = \{a, e, i, o, u\} \)

Example 1.7

Let \( P \) be the set of all prime numbers less than 10. Write \( P \) using listing method.
Solution:
The prime numbers less than 10 are 2, 3, 5 and 7.
Therefore, \( P = \{2, 3, 5, 7\} \).

Note that the elements of such types of sets can be completely or partially listed and the elements follow certain pattern.

Example 1.8
If \( B \) is the set of natural numbers less than 100, then write \( B \) using listing or roster method.

Solution:
The natural numbers less than 100 are numbers from 1 to 99. Instead of writing all the 99 numbers, you can list the first three numbers to indicate that the numbers follow a certain pattern followed by three dots to indicate that the numbers are continuing up to the last number, which is 99.

Thus, \( B = \{1, 2, 3, \ldots, 99\} \) and this is a partial listing method.

Example 1.9
If \( O \) is the set of odd natural numbers, then write \( O \) using listing or roster method.

It is not possible to list all the odd natural numbers, but you can list the first three odd natural numbers followed by three dots to indicate that the listing continues as indicated in the pattern.

Therefore, \( O = \{1, 3, 5, \ldots\} \). It is again a partial listing method.

Verbal Method
There are times when it is not practical to list all the elements of a set. In this case, it is better to describe the set using verbal method. The rule that the elements follow can be given in the braces or without braces.

Example 1.10
Describe each of the following sets

a. The set of letters in the English alphabet.

b. The set of Grade 7 students in Ethiopia in 2015 E.C.

Solution:

a. There are 26 letters in the English alphabet and instead of listing all the English alphabets, you can write the set as \( \{\text{the English alphabets}\} \).
b. It is very difficult to list all Grade 7 students in Ethiopia in 2015 E.C. and the better way to write it is as \{Grade 7 students in Ethiopia in 2015 E.C.\}.

Exercise 1.2

1. Describe each of the following sets by the listing method.
   a. The set of all subjects that a Grade 7 student is learning.
   b. The set of capital cities of the regional states in Ethiopia.
   c. The set of prime numbers between 4 and 20.
   d. The set of even natural numbers.
   e. The set of all odd natural numbers less than 100.

2. Describe each of the following sets by a verbal method.
   a. \{1, 2, 3, \ldots, 100\}
   b. \{1, 3, 5, 7, \ldots\}
   c. \{10, 20, 30, 40, \ldots\}
   d. \{January, February, March, \ldots, December\}
   e. \{2, 3, 5, 7, 11, \ldots\}

1.3 Some Special Types of Sets

Activity 1.3

Describe each of the following sets.

a. A is the set of triangles with four sides.

b. B is the set of natural numbers less than 20.

c. C is the set of even natural numbers.

What differences do you observe between the sets A, B and C?

From your responses in Activity 1.3, observe that:

1. set A has no element;

2. set B has 9 elements and

3. the number of elements in set C is not finite.
UNIT 1: INTRODUCTION TO SETS

Definition:
A set which contains no element is called **empty set (null or void set)**. Empty set is denoted by $\emptyset$ or $\{\}$. 

Example 1.11
Which of the following sets are empty sets.

a. A is the set of whole numbers that are odd and even.

b. B is set of whole numbers that are even and prime.

c. D is the set of all weeks with 8 days.

Solution:

a. There is no whole number which is both even and odd. So A is empty set, that is, $A = \{\}$. 

b. The only whole number that is both even and prime is 2. Therefore, $B = \{2\}$.

c. Every week has 7 days and there is no week of 8 days. Therefore, $D$ is empty set, that is $D = \emptyset$.

Definition:
A set $S$ is said to be a finite set if $S$ contains exactly $n$ elements for some positive integer $n$ or $S = \emptyset$. A set that is not finite is called infinite set.

Notation:
If $S$ is a finite set, then the number of elements in $S$ is denoted by $n(S)$.

Example 1.12
Identify each of the following sets as finite or infinite.

a. A is the set of all months of a year with 32 days.

b. B = \{1, 2, ..., 100\}.

c. M is the set of all natural numbers that are multiples of 4.

Solution:

a. There is no month of a year with 32 days. Therefore, $A = \{}$ and hence $A$ is a finite set, with $n(A) = 0$.

b. B contains 100 elements. Thus, B is a finite set and $n(B) = 100$.

c. $M = \{4, 8, 12, \ldots\}$ and $M$ does not contain finite elements. Therefore, $M$ is an infinite set.
**Exercise 1.3**

Identify each of the following sets as finite set, infinite set or empty set.

- **a.** A is the set of integers greater than 10.
- **b.** B is the set of natural numbers less than 100.
- **c.** C is the set of natural numbers less than 0.
- **d.** S is the set of all elementary school students in Ethiopia.

### 1.4 Relationships between Sets

#### Equality of Sets

**Activity 1.4**

A mathematics teacher asked her students to write the set of primary colors using roster notation. She received two different answers from two different students as shown below. Melkam write \( S = \{ \text{red, yellow, blue} \} \) and Berhanu write \( B = \{ \text{blue, red, yellow} \} \). Which student is correct?

From your responses to Activity 1.4, observe that, both students wrote the three basic colors and the only difference between the two answers is the order of writing the colors.

In writing a set, order is not important and the two sets are the same and we say that the two sets \( S \) and \( B \) are equal. We write this relations as \( S = B \).

**Definition:**

Two sets \( A \) and \( B \) are said to be equal, written as \( A = B \), if they have the same number of elements, and their elements are the same. The order in which the elements appear in the set is not important.

**Example 1.13**

Which of the following sets is equal to a set \( A \) which is the set of whole numbers less than 5?

- **a.** \( B = \{2, 0, 3, 1, 4\} \)
- **b.** \( C = \{a, e, i, o, u\} \)
- **c.** \( D = \{1, 2, 3, 4, 5\} \)
Solution:
The whole numbers less than 5 are 0, 1, 2, 3 and 4. Thus, any set containing only these numbers is equal to A, otherwise it is not equal to A.
So from the given lists, A = B, but A \neq C and A \neq D.

 Subset

Activity 1.5

What relationships do you observe between each of the following pairs of sets?

a. \( A = \{1, 2, 3, 4, 5\} \) and \( B = \{3, 1, 2, 5, 4, 6\} \),

b. C is the set of the first five English alphabets and \( D = \{0, 1, 2, 3, 4\} \).

From your responses in Activity 1.5, observe that every element of A is an element of B and in this case we say that A is a subset of B, but 6 is in B, which is not in A.

Definition:

Given two sets A and B if every element of A is an element of B, then we say that A is a subset of B, and we denote this relation by \( A \subseteq B \).

Example 1.14

1. From your knowledge on numbers, recall that every natural number is a whole number and every whole number is an integer.
   Therefore, if \( N \) is the set of natural numbers, \( W \) is the set of whole numbers, then we have the following relationships.
   \( N \subseteq W \)

2. Let \( A \) be the set of all multiples of 6, \( B \) be the set of all multiples of 2 and \( C \) be the set of all multiples of 3.
   As any multiple of 6 is a multiple of both 2 and 3, every element of A is an element of B and an element of C.
   Therefore, \( A \subseteq B \) and \( A \subseteq C \).

Note:

For any set ,

a. Empty set is a subset of every set or \( \emptyset \subseteq A \)
b. Every set is the subset of itself or \( A \subseteq A \).
Example 1.15
List all subsets of the set \( A = \{a, b, c\} \). How many are there?

Solution:
The subsets of \( A \) are \( \{a\}, \{b\}, \{c\}, \{a, b\}, \{b, c\}, \{a, c\}, \{a, b, c\} \) and \( \emptyset \).
There are eight subsets of the set \( A = \{a, b, c\} \).

Example 1.16
List all subsets of the set \( N = \{0, 1, 2, 3\} \). How many are there?

Solution:
The subsets of \( N \) are \( \{0\}, \{1\}, \{2\}, \{3\}, \{0, 1\}, \{0,2\}, \{0, 3\}, \{1, 2\}, \{1, 3\}, \{2, 3\}, \{0, 1, 2\}, \{0, 2, 3\}, \{1,2,3\}, \{0, 1, 3\}, \{0, 1, 2, 3\} \) and \( \emptyset \).
There are 16 subsets of the set \( N = \{0, 1, 2, 3\} \).

Note:
In Example 1.15, set \( A \) has three (3) elements and eight (8) subsets. In Example 1.16, set \( N \) has four (4) elements and 16 subsets.
Thus, the number of subsets of a set \( A \) with \( n \) elements is \( 2^n \).

Union and Intersection of Sets
The set containing all the elements in both \( A \) and \( B \) is the intersection of \( A \) and \( B \), and the set containing all the elements in either \( A \) or \( B \) is the union of \( A \) and \( B \).

Activity 1.6
Let \( A = \{1, 2, 3, 4\} \) and \( B = \{0, 2, 4, 6, 8\} \).

a. Find the set containing the elements that are in both \( A \) and \( B \).

b. Find the set containing the elements that are in either \( A \) or \( B \).

The set containing all the elements in both \( A \) and \( B \) is the intersection of \( A \) and \( B \), and the set containing all the elements in either \( A \) or \( B \) is the union of \( A \) and \( B \).

Definition:
Let \( A \) and \( B \) be two sets.

1. The intersection of two sets \( A \) and \( B \), denoted by \( A \cap B \) (read as \( A \) intersection \( B \)), is the set of all elements that are elements of both sets \( A \) and \( B \).

2. The union of two sets \( A \) and \( B \), denoted by \( A \cup B \) (read as \( A \) union \( B \)), is the set of all elements that are in \( A \) or in \( B \).
UNIT 1: INTRODUCTION TO SETS

Example 1.17

a. Let \( A = \{1, 2, 3, 4\} \) and \( B = \{0, 2, 4, 6, 8\} \). Then find \( A \cap B \) and \( A \cup B \).

b. Let \( A = \{a, e, i, o, u\} \) and \( B = \{a, b, c, d, e\} \). Then find \( A \cap B \) and \( A \cup B \).

Solution:

a. \( A \cap B = \{2, 4\} \) and \( A \cup B = \{0, 1, 2, 3, 4, 6, 8\} \).

b. \( A \cap B = \{a, e\} \) and \( A \cup B = \{a, b, c, d, e, i, o, u\} \).

Venn Diagrams

There is another way to represent sets, which is with a visual tool called a Venn diagram. In Venn diagrams sets are represented by shapes; usually circles or oval shapes and the elements of a set are labeled within the shape.

Example 1.18

Let \( V \) be the set of vowel letters in the English alphabet. Draw a Venn diagram to represent set \( V \) and indicate all elements in the set.

Solution:

\( V = \{a, e, i, o, u\} \).

First draw a circle and level the elements in \( V \) as in the following diagram.

Example 1.19

Given set \( B \) is the set of primary colors. Draw a Venn diagram to represent the set \( B \), and indicate all elements in the set.

Solution:

The set \( B \) is given by \( B = \{\text{red}, \text{yellow}, \text{blue}\} \).

Draw a circle, label it by \( B \) and put the elements in the circle as in the following diagram.

In each of the examples above, we used a Venn diagram to represent a given set pictorially. Venn diagrams are also useful to show relationships between sets.
**Example 1.20**

Let $A = \{1, 2, 3, 4\}$ and $B = \{3, 4, 5, 6, 7\}$. Draw and label a Venn diagram to show $A \cap B$.

**Solution:**

You need to find the elements that are common in both sets.

First draw two overlapping ovals as shown in the figure below. Elements that are common to sets $A$ and $B$, 3 and 4 in this case, will be placed in the region that is common for both ovals.

![Venn diagram](image)

**Example 1.21**

Let $X = \{a, b, 1, 2\}$ and $Y = \{1, 2, 3, d, e\}$. Draw and label a Venn diagram to represent $X \cup Y$.

**Solution:**

The union of $X$ and $Y$ is any element in $X$ or in $Y$ or in both $X$ and $Y$.

So $X \cup Y = \{1, 2, 3, a, b, d, e\}$.

Draw two circles that have intersection points as shown below.

The shaded region shows the union of these two sets.

![Venn diagram](image)

**Venn diagram and Relationships between Sets**

Given two sets $A$ and $B$, there are different possibilities about the relationships between these sets. Some of these relationships are

a. $A \subseteq B$; $A$ is a subset of $B$;

b. $A = B$; $A$ and $B$ are equal sets.

**Example 1.22**

Given $A = \{1, 2, 4\}$ and $B = \{1, 2, 3, 4, 5\}$, find the relationship between $A$ and $B$. 

![Venn diagram](image)
Solution:
As you can see, all the elements in A are elements of B.
On the other hand, 3 and 5 are elements of B, but they are not elements of A.
To represent this relationship using a Venn diagram, first draw two circles, one inside the other and represent the outer circle by B and the inner circle by A.
Put the common elements of A and B, in the inner circle and the elements that are only in B, but not in A in the part of outer circle, as shown in the diagram below.

Example 1.23
Represent the sets $A = \{1, 2, 3, 4, 5\}$ and $B = \{3, 1, 2, 5, 4\}$ using a Venn diagram.

Solution:
You can see that A and B are equal sets, that is $A = B$.
Draw a circle and represent it by A and B.
Put the elements of A and B inside as shown in the diagram below.

Exercise 1.4

1. Which of the following sets are equal?
   a. $A = \{a, b, c, d\}$
   b. $B = \{d, e, a, c\}$
   c. $C = \{d, b, a, c\}$
   d. $D = \{a, a, d, e, c, e\}$
   e. $E = \text{All integers between 1 and 10}$
   f. $F = \text{Even integers from 1 to 10}$
   g. $G = \text{Odd integers from 1 to 10}$
   h. $H = \text{Multiples of 2 from 1 and 10}$

2. Let $A = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$, $B = \{2, 4, 6, 8\}$, $C = \{1, 3, 4, 5, 7\}$. Find:
   a. $A \cap B$
   b. $B \cap A$
   c. $B \cup C$
   d. $C \cup B$

3. Let $A = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$, $B = \{1, 2, 3, 4, 5\}$, $C = \{0, 2, 4, 6, 8\}$, $D = \{3, 6, 9\}$. Then identify each of the following as true or false and give your reasons.
   a. $B \subseteq A$
   b. $C \subseteq A$
   c. $B \subseteq C$
   d. $D \subseteq B$
   e. $D \subseteq A$
   f. $C \cap A$. 
4. List all the subsets of each of the following sets.
   a. \( A = \{a, b, c, d, e\} \)
   b. \( B = \) The prime factors of 60.

5. Consider the Venn diagram representing the sets R, E and N given below. Then find each of the following.
   a. \( (R \cup E) \cup N \)
   b. \( R \cup E \)
   c. \( R \cup N \)
   d. \( E \cup N \)
   e. \( R \cap (E \cap N) \)
   f. \( R \cap N \)
   g. \( R \cap E \)
   h. \( E \cap N \)

Unit summary

1. A set is a well-defined collection of objects and the objects of a set are called elements of the set. Elements of a set are also called members of the set.

2. Sets are described by listing method or verbal method.

3. A set that has no element is called an empty set and empty set is denoted by \( \emptyset \) or \( \{\} \).

4. A set \( A \) is called finite if and only if it is the empty set or has exactly \( n \) elements, where \( n \) is a natural number. A set that is not finite is called an infinite set.

5. Two sets \( A \) and \( B \) are equal if every element of one is an element of the other.

6. A set \( A \) is a subset of \( B \) if every element of \( A \) is an element of set \( B \).

7. For any two sets \( A \) and \( B \),
   a. The union of \( A \) and \( B \), denoted by \( A \cup B \), is the set of all elements that are members of either \( A \) or \( B \).
   b. The intersection of \( A \) and \( B \), denoted by \( A \cap B \), is the set of all elements that are members of both \( A \) and \( B \).
Review Exercises

1. Which of the following are sets?
   a. The collection of all tall students in your class.
   b. The collection of all natural numbers divisible by 3.
   c. The collection of all students in your school.
   d. The collection of all intelligent students in Ethiopia.
   e. The collection of all subsets of the set \{1, 2, 3, 4, 5\}.

2. Which of the following represent equal sets?
   a. \(A = \{a, b, c, d, e\}\)
   b. \(B = \{a, e, i, o, u\}\)
   c. \(C = \) The first five letters in the English alphabet.
   d. \(D = \) The vowel letters in the English alphabet.

3. If \(X = \{a, b, c, d, e, f, g, h, i, j\}\), \(Y = \{b, d, f, h\}\) and \(Z = \{a, b, e, f, g, h\}\), then find each of the following sets:
   a. \(X \cap Y\)
   b. \(Y \cap X\)
   c. \(Y \cup Z\)
   d. \(Z \cup Y\)
   e. \((X \cup Y) \cup Z\)
   f. \((X \cap Y) \cap Z\)

4. Identify each of the following as finite or infinite.
   a. \(A = \) All integers less than 10.
   b. \(B = \) All students in your class who are older than 20 years of age.
   c. \(C = \) All natural numbers less than 10.

5. Use the Venn diagram given below to find each of the following set relations.

\[
\begin{array}{c}
\text{a. } A \cap (B \cap C). \\
\text{b. } A \cup B \\
\text{c. } A \cap B \\
\text{d. } A \cup C \\
\text{e. } A \cup (B \cap C). \\
\text{f. } B \cap C \\
\text{g. } A \cap C \\
\text{h. } B \cup C
\end{array}
\]
UNIT - 2

INTEGERS

Learning Outcomes:

After completing this unit, you will be able to:

- Understand the concept of integers.
- Represent integers on a number line.
- Perform the operations of addition, subtraction, multiplication and division on integers.
- Identify the commutative, associative and distributive properties of operations on integers.
- Apply operation of integers in the real-life situation.

Key Terms

- Natural number
- Integer
- Odd and even integer
- Additive inverse
- Quotient
- Factors
- Commutative
- Distributive properties
- Whole number
- Predecessor and successor of an integer,
- Prime and composite number
- Dividend, divisor,
- Multiples
- Identity element
- Associative
UNIT 2: INTEGERS

Introduction

In the previous grades, you have learnt about natural numbers (\(\mathbb{N}\)) and whole numbers (\(\mathbb{W}\)). These numbers and their opposites form a bigger set of numbers known as integers. In this unit, you will learn about integers, operations on the set of integers and their basic properties. Also, you will practice basic operations on integers to solve real life practical problems.

2.1 Revision on Natural Numbers and Whole Numbers

From your study of numbers in the previous grades what do you remember about natural numbers and whole numbers?

You have learned numbers such as 1, 2, 3, 4, 5, 6, ... Such numbers are called natural numbers. Since they are used to count objects, natural numbers are also called counting numbers.

You have also learned numbers such as 0, 1, 2, 3, 4, ... Such numbers are called whole numbers.

Within natural numbers you have learnt also odd and even, prime and composite natural numbers. The following table contains the first 100 natural numbers. If you examine the table carefully you can identify odd and even, prime and composite natural numbers.

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
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<td>97</td>
<td>98</td>
<td>99</td>
<td>100</td>
<td></td>
</tr>
</tbody>
</table>

Activity 2.1

Copy the table above and do the following activities.

a. Which columns contain only odd natural numbers?

b. Which columns contain only even natural numbers?

c. Can you observe a column containing both even and odd natural numbers?

d. Follow the following procedure:
Circle 2. Cross all multiples of 2. Multiples of 2 are not prime numbers.
Circle 3. Cross all multiples of 3. Multiples of 3 are not prime numbers.
Repeat the process with other numbers 5, 7, 11, 13..., what do you observe?
   i. List all the circled numbers. What are these numbers?
   ii. List all the crossed numbers. What are these numbers?

Example 2.1
Consider the following numbers and answer questions that follow
   2, 8, 11, 20, 21, 23, 25, 40, 41, 45, 55, 57, 59, 73, 100

   a. Which of them are Odd numbers? Which are even numbers?
   b. Which of them are prime numbers? Which are composite numbers?
   c. Which of the numbers in the list above are with their successor or predecessor?

Solution:
   a. Odd numbers: 11, 21, 23, 25, 41, 45, 55, 57, 59, 73
      Even numbers: 2, 8, 20, 40, 100
   b. Prime numbers: 2, 11, 23, 41, 59, 73
      Composite numbers: 8, 20, 21, 25, 40, 45, 55, 57, 100
   c. 20 is a predecessor of 21, and 21 is the successor of 20. 40 is a predecessor of 41, and 41 is the successor of 40.

Note:
1. A natural number having only two factors, namely 1 and itself, is a **prime number**.
2. A natural number having more than two factors is a composite number.
3. A natural number that can be divided by 2 without a remainder is even and a natural number that can be divided by 2 with a remainder is odd natural number.
4. The number that comes just before a number is called the predecessor.
5. The number that comes just after a number is called the successor.
   So, the predecessor of a number is 1 less than the given number, and the successor of a number is 1 more than the given number.
UNIT 2: INTEGERS

Exercise 2.1

1. Identify the following numbers
   a. which are prime?
   b. which are composite?

   \[21, 27, 35, 97, 83, 021\]

2. Fill the blank spaces and make true complete statements.
   a. _____ is between _____ and ____. Therefore, _____ is the predecessor of 101.
   b. _____ is between _____ and ____. Therefore, _____ is the successor of 96.
   c. _____ is between _____ and ____. Therefore, _____ is the predecessor and _____ is the successor of ______.
   d. If “n” is a natural number, then its successor is ______ and its predecessor is ______

3. Has 1 a predecessor in natural numbers? What about the predecessor of 0 in whole numbers?

4. What is the successor of the largest 2-digit number?

5. What is the predecessor of the smallest 3-digit number?

6. Answer the following questions
   a. Is 1 a prime number?
   b. Is 1 a composite number?
   c. Explain your answer on a and b

Greatest Common Factor (GCF) and Least Common Multiple (LCM) of Natural Numbers

Activity 2.2

Take any two natural numbers. Then,

a. List all factors of the two numbers.
   Identify the common factors.
   Which common factor is the greatest?
Among the common factors of two or more numbers there will always be a largest number, which is called the greatest common factor (GCF). Among the common multiples of two or more numbers there will always be a smallest natural number, which is called the least common multiple (LCM).

**Example 2.2**

Find the greatest common factor (GCF) of 18 and 27.

**Solution:**

The factors of 18 are: 1, 2, 3, 6, 9, and 18.

The factors of 27 are: 1, 3, 9, and 27.

The common factors of 18 and 27 are: 1, 3, and 9.

Therefore, the GCF of 18 and 27 is 9 or GCF(18,27) = 9

**Example 2.3**

Find the least common multiple (LCM) of 8 and 12.

**Solution:**

Multiples of 8 are: 8, 16, 24, 32, 40, 48, etc.

Multiples of 12 are: 12, 24, 36, 48, 60, etc.

The common multiples of 8 and 12 are: 24, 48, etc.

Therefore, the LCM of 8 and 12 is 24 or LCM(8,12) = 24

**Example 2.4**

Find the GCF of 14 and 15

**Solution:**

The factors of 14 are: 1, 2, 7 and 14.

The factors of 15 are: 1, 3, 5, and 15.

The common factor of 14 and 15 is 1.

Therefore, GCF(14,15) = 1
UNIT 2: INTEGERS

Note:

• If the GCF of two natural numbers is 1, then the numbers are said to be relatively prime numbers. Therefore, 14 and 15 are relatively prime numbers.
• The GCF and LCM of two or more natural numbers can also be obtained by using prime factorization.
  » The GCF of two or more numbers can be obtained by multiplying each common prime factor the minimum number of times it occurs in each of the numbers.
  » The LCM of two or more numbers can be obtained by multiplying each prime factor the maximum number of times it occurs in each of the numbers.

Example 2.5

Find the GCF and LCM of: 60 and 72

Solution:

Prime factorization: \(60 = 2^2 \times 3 \times 5\) and \(72 = 2^3 \times 3^2\)

i. Identify the common factors:

\[
60 = 2^2 \times 3 \times 5 \\
72 = 2^2 \times 3 \times 2 \times 3 \\
\]

Thus, \(2 \times 2 \times 3\) is the multiple of the common factors.

Therefore, \(GCF(60,72) = 2 \times 2 \times 3 = 2^2 \times 3 = 12\)

ii. Identify all the common and non-common factors

\[
60 = 2^2 \times 3 \times 5 \\
72 = 2^2 \times 3^2 \times 2 \times 3 \\
\]

Thus, \(2^2 \times 2 \times 3 \times 3 \times 5\) is the product of the common and not common factors

Therefore, \(LCM(60,72) = 2 \times 2 \times 2 \times 3 \times 3 \times 5 = 2^3 \times 3^2 \times 5 = 360\)

Note:

• A composite number can be written as a product of prime numbers in exactly one way if you ignore the order of the factors. This product is called the prime factorization of the number.
• The Greatest Common Factor (GCF) of two or more natural numbers is the largest factor which is common to each number.
UNIT 2: INTEGERS

- The Least Common Multiple (LCM) of two or more natural numbers is the smallest non zero multiple which is common to each number.

Exercise 2.2

1. List all factors of the pairs of numbers given below and identify the common factors.
   a. 4 and 6
   b. 10 and 12
   c. 6 and 8
   d. 7 and 11
   e. 8 and 9

2. List some multiples of each pair of numbers and identify at least two common multiples.
   a. 4 and 6
   b. 8 and 9
   c. 7 and 11
   d. 6 and 8
   e. 10 and 12

3. Write the prime factorization of each of the following numbers.
   a. 78
   b. 447
   c. 222
   d. 19
   e. 300
   f. 333

4. Find the GCF and LCM of each of the following pairs of numbers.
   a. 4 and 6
   b. 24 and 25
   c. 8, 12 and 18
   d. 8, 20, 64 and 96

5. Find two numbers p and q for which their GCF is 7 and their LCM is 210. How many possible values of p and q do you get? List them.

Problem Solving

The expression $n^2 + n + 11$ may result in a prime number when n is replaced with whole numbers. Evaluate $n^2 + n + 11$ for whole numbers 0, 1, 2, 3, ..., 10. Are the resulting numbers prime or composite?

2.2 Introduction to Integers

Whole numbers are useful in solving many real-life problems. However, there are many other situations that cannot be expressed using whole numbers alone. For example, describing temperature below zero, altitude below sea level, credits or loss of money and so on cannot be expressed by whole numbers. In this subsection, we study operation with zero, natural numbers and negative numbers.
Activity 2.3

Look at the table below showing the highest and the lowest average temperature records of five towns. Answer the following questions based on the table below.

<table>
<thead>
<tr>
<th>Town</th>
<th>Highest temperature in °C</th>
<th>Lowest Temperature in °C</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>27 above 0</td>
<td>13 above 0</td>
</tr>
<tr>
<td>B</td>
<td>12 above 0</td>
<td>2 below 0</td>
</tr>
<tr>
<td>C</td>
<td>18 above 0</td>
<td>3 below 0</td>
</tr>
<tr>
<td>D</td>
<td>10 above 0</td>
<td>5 below 0</td>
</tr>
<tr>
<td>E</td>
<td>8 above 0</td>
<td>0</td>
</tr>
</tbody>
</table>

a. Which town had the highest temperature?

b. Which town had the lowest temperature?

c. Find the difference between the highest and the lowest temperatures of each town.

Temperatures below 0 are negative temperatures and represented by “−” sign. Temperatures above 0 are positive temperatures and represented by “+” sign. Therefore, in the table above, 2°C below 0 is -2; 3°C below 0 is -3; and 5°C below 0 is -5. We can also say that 8°C above 0 is +8; 10°C above 0 is +10 and the like.

Example 2.6

Look at the association of the numberings on the thermometer and the number line. What similarities do you observe?

Thus, the numbers associated with the thermometer are integers. Denoted by a symbol \( \mathbb{Z} \) and defined as

\[ \mathbb{Z} = \ldots, -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5, \ldots \]

Integers that are greater than zero are positive integers \( (\mathbb{Z}^+) \). Integers less than zero are negative integers \( (\mathbb{Z}^-) \). Zero is neither negative nor positive. You can show positive and negative integers on a number line.
The number 0 is neither positive nor negative

\[ \mathbb{Z}^+ = \{1, 2, 3, 4, 5, 6, 7, \ldots\}\]
\[ \mathbb{Z}^- = \{\ldots, -5, -4, -3, -2, -1\}\]

Positive and negative integers can also be associated with credit and debit, deposit and withdrawal, profit and loss, etc. For example, if someone’s account is credited by 200 Birr, it means +200, but, if someone’s account is debited by 200 Birr, it means -200. If someone’s account neither debited nor credited means no change in balance.

**Exercise 2.3**

1. What makes integers different from natural numbers and whole numbers?
2. Are there integers that are not whole numbers? Why?
3. Are there integers that are not natural numbers? Why?
4. What do you conclude about the relationships among \(\mathbb{N}\), \(\mathbb{W}\), and \(\mathbb{Z}\)?
5. Represent each of the following situations by an integer.
   a. A loss of 6 Birr
   b. A deposit of 10 Birr
   c. 16 °C below 0 °C
   d. 10 steps walking backward
   e. Increment of 2 units
   f. 116 meters below sea level
   g. Withdrawal of 100 Birr
   h. A profit of 10 Birr
   i. Loss of 2 points in a game
   j. 38 degree below freezing point

**Activity 2.4**

Explain how the following paired situations are related.

a. -18 and +18.
b. +9 and -9.
c. A profit of 100 Birr and loss of 100 Birr.
Two integers in different directions on the number line and at equal distance from 0 are opposite integers.

**Example 2.7**

Find the opposite of -5, 24 and 212.

**Solution:**

The opposite of -5 is 5, because -5 and 5 are at equal distances from 0 on a number line and they are in different directions.

- -24 is the opposite of 24,
- -212 is the opposite of 212.

For each integer, there is another integer, called its opposite, such that their sum is zero. We can show opposites on a number line. For example -5 and its opposite is shown below.

![Number Line with Opposites](image)

**Definition:**

Two numbers situated on the number line symmetrically opposite to 0 are called opposite numbers. The number 0 is opposite to itself. The opposite of an integer a (a ≠ 0) is -a, and vice versa.

**Exercise 2.4**

1. On the number line given below determine the integers represented by the letters.

![Number Line with Letters](image)
2. Find the opposite of each of the following integers.
   a. 3  
   b. 12,345  
   c. -95  
   d. -106  
   e. 4,545

3. Find the integer x, that satisfies each of the following condition.
   a. 174 + x = 0  
   b. -351 + x = 0  
   c. x + (-20150) = 0  
   d. -36 + x = 14

4. Estimate the position of the numbers -250, -115, -10, 450, 125, 20, 805, 345, 675 on the number line given below. Estimate the position of the opposite of each of the numbers on the given number line.

![Number Line]

2.3 Comparing and Ordering Integers

**Activity 2.5**

1. Use a number line to answer the questions. Complete the following blank spaces using the words ‘left’ or ‘right’ and then write ‘<’ or ‘>’ to make a true statement.
   a. -4 is to the _______ of 9, so -4 _______ 9  
   b. 9 is to the _______ of 6, so 9 _______ 6  
   c. -10 is to the _________ of -12, so -10 _________ -12  
   d. 3 is to the _________ of -7, so 3 _________ -7

2. Consider the following numbers: -45, 101, 33, -27, 89, 205, 1001.
   a. Rewrite them in an ascending order.  
   b. Rewrite them in descending order.

Comparing and ordering of numbers can be illustrated with a number line. For any two numbers on a number line, the number to the left is less than the number to the right. As you go towards left on a number line, you will get integers in descending order and as you go towards the right you will get integers in ascending order.

**Example 2.8**

Abeba and Hailu bought books that cost 100 Birr each. Abeba paid 89 Birr and Hailu paid 85 Birr in cash. Abeba has 11 Birr and Hailu has 15 Birr credit yet to pay to the book store.
seller. Who has less balance of money?

**Solution:**

Abeba’s balance is -11 Birr and Hailu’s balance is -15 Birr.

You can use a number line to compare -11 and -15. As you can see on the number line below, -15 is to the left of -11. Thus, -15 < -11. Therefore, Hailu has less balance than Abeba.

![Number Line](image)

**Note:**

- For any two integers a and b, only one of the following holds true.
  - a < b or a > b or a = b is called trichotomy property of numbers.

---

**Exercise 2.5**

1. Compare the pair of integers by writing the signs ‘>’, ‘<’, or ‘=’ on the blank spaces.
   - a. 6 _______ -66
   - b. -7 _______ -700
   - c. -999 _______ -99
   - d. 45 _______ 54
   - e. -573 _______ -375
   - f. 0 _______ -45

2. Any negative integer is less than any positive integer. Why?

3. Arrange each of the following integers from the smallest to the largest.
   - a. 56, -56, -40, 75, -105, -501
   - b. -1, 0, -11, -101, -111, 101, 11
   - c. 75, -3, -4, 12, 0, 9, -10

4. The following table shows substances with their temperature that solidify (turn to solid). Arrange the elements starting with the liquid which has the lowest solidifying temperature to the highest.

<table>
<thead>
<tr>
<th>Substances</th>
<th>Ammonia</th>
<th>Mercury</th>
<th>Alcohol</th>
<th>Aniline</th>
<th>Water</th>
<th>Carbone Dioxide</th>
<th>Glycerin</th>
<th>Lead</th>
</tr>
</thead>
<tbody>
<tr>
<td>Temperature</td>
<td>-78°C</td>
<td>-39°C</td>
<td>-97°C</td>
<td>-6°C</td>
<td>0°C</td>
<td>-78°C</td>
<td>-16°C</td>
<td>327°C</td>
</tr>
</tbody>
</table>
5. If 0 is the largest number in a group of five integers, what can you conclude about the other four integers? What is your answer, if the word ‘largest’ is replaced by the ‘smallest’?

6. What is the largest negative integer? What about the smallest positive integer?

7. Consider the following list of minimum temperatures (°F) of a town in Ethiopia recorded in each Month of a year.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Temp.</td>
<td>-2</td>
<td>10</td>
<td>25</td>
<td>24</td>
<td>26</td>
<td>31</td>
<td>48</td>
<td>8</td>
<td>-12</td>
<td>-18</td>
<td>-15</td>
<td></td>
</tr>
</tbody>
</table>

a. Write the temperatures from the least to the highest (in an ascending order).

b. On which month is the coldest temperature recorded? The warmest temperature recorded?

### Problem Solving

1. If points A, B, C and D represent integers on a number line, using the following clues, arrange the integers in their order.
   a. D is the least of integers
   b. A and D are opposite integers
   c. C is closer to D than it is to B
   d. C is a positive integer.

2. **Project work:** Record the daily temperature of different Ethiopian towns presented by Ethiopian Television or radio, and then perform the following activities.
   a. Mark the temperature on the number line
   b. Which town is the coldest?
   c. Which town is the warmest?
   d. Arrange the towns in increasing temperatures.
2.4 Addition and Subtraction of Integers

2.4.1 Addition of Integers

Activity 2.6

Thunderstorms are made of both positive and negative electrical charges. There are extra negative charges (electrons) at the bottom of a thundercloud, and extra positive charges (protons) at the top.

1. What is the charge at the top of a cloud where there are more protons than electrons?

2. What is the charge at the bottom of a cloud where there are more electrons than protons?

Combining positive and negative electrical charges in a thunderstorm is similar to adding integers.

When a positive and a negative integer are added, the sum will be positive or negative or zero depending on the sign of the number that is farthest from zero. To add integers, you can use counters or a number line.

Example 2.9

Adding integers with the same sign

Find

\[ +3 + (+5) \]

Solution:

Method: 1 Using counters

Positive numbers are represented by positive counters and negative numbers are represented by negative counters.

So, \( +3 + (5) = +8 \) or 8
When using counters for addition of integers of the same sign, the sum of the integers is the number of counters all together.

**Method: 2 Using a number line**

- Start at 0, Move 3 units right to show +3.
- From there, move 5 units right to show +5.
  
  So, \( +3 + (+5) = +8 \) or 8.

\[ \begin{array}{c}
-1 & 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 \\
\hline
+3 & & & & & & & & & & & \\
+5 & & & & & & & & & & & \\
\end{array} \]

\[ 3 + 5 = 8 \]

b. \(-3 + (-5)\)

**Method I: Using Counters**

\(-3 + (-5) = -8\)

**Method II: Using Number line**

\(-3 + (-5) = -8\)

- Start at 0, move 3 units left to show -3.
- From there, move 5 units to the left to show -5
  
  so, \(-3 + (-5) = -8\)

---

**Note:**

- The sum of two positive integers is always positive.
- The sum of two negative integers is always negative.
- To add two negative integers, add the two numbers without the negative sign, and then give a negative sign for the result.

To add two integers with different signs, it is necessary to remove any zero pairs (opposites). A **zero pair** is a pair of counters that includes one positive counter and one negative counter.
Example 2.10

Adding integers with different signs

Find: $-3 + 5$

**Solution:**

**Method: 1 Using counters**

- Place three negative counters and five positive counters side by side.
- Pair the positive and the negative counters, and then remove all zero pairs.
- Only two positive counters remain.

**Method: 2 Using a number line**

Start at 0,

- Move 3 units left to show $-3$.
- From there, move 5 units right to show $+5$.
  
  So, $-3 + (+5) = +2$ or 2.

**Note:**

1. The sum of a positive integer and a negative integer is sometimes positive, sometimes negative, and sometimes zero.
2. To add integers with opposite sign, compare the two numbers without their signs, subtract the smaller from the larger number, and then give the sign of the larger number to the result.

Example 2.11

Find: $-45 + 32$

**Solution:**

To add $-45 + 32$, first take the numbers without their signs, subtract 32 from 45 ($45 - 32 = 13$).

Since the sign of 45 is negative, their sum is negative.

Therefore, $-45 + 32 = -13$. 
Exercise 2.6

1. Use number line or counters to add.
   a. 1 + 5
   b. -3 + (-2)
   c. -5 + (-4)
   d. -5 + (-7)
   e. -10 + (-4)
   f. -14 + (-16)
   g. 23 + 38

2. Use number line or counters to add, if necessary.
   a. 7 + (-5)
   b. -6 + 4
   c. -1 + 8
   d. 6 + (-7)
   e. -15 + 19
   f. 10 + (-12)
   g. -27 + 27
   h. 15 + 9 + (-9)

3. In a certain time the temperature of a room was -3°C. An hour later, it drops 6°C and 2 hours later, it rises 4°C. Write an addition sentence to describe this situation. Then find the sum and explain its meaning.

Properties of Addition on Integers

Activity 2.7

Compare the following paired values.

a. -9 + 7 and 7 + (-9)
   c. 37 + 0 and 0 + 37
b. (-2 + 5) + 3 and -2 + (5 + 3)
   d. 43 + (-43) and -43 + 43

What do you understand from the above paired values?

From the above activity, you can understand that, when you add two or more integers, their sum is the same regardless of the order or the grouping of the addends. Based on your work on the activity, the following properties of addition on integers can be generalized. These properties can help you add three or more integers.

Properties of Addition on Integers

1. Commutative property of addition.
   For any two integers a and b; a + b = b + a.
   For example, 3 + (-2) = -2 + 3 = 1.

2. Associative property of addition.
   For any three integers a, b and c; (a + b) + c = a + (b + c).
   For example, (-3 + 5) + (-8) = -3 + (5 + (-8)) = -6.
UNIT 2: INTEGERS

3. Property of 0 for addition.
   For any integer a, \( a + 0 = a = 0 + a \).
   0 is called the identity element for addition on integers.
   For example, \(-4 + 0 = -4 = 0 + (-4)\).

4. Inverse property of addition
   For any integer a, \( a + (-a) = 0 = -a + a \).
   -a is the additive inverse of a or vice versa.
   For example, \(-9 + 9 = 0 = 9 + (-9)\).

Example 2.12

Find: \(-4 + (-12) + 4\).

Solution:

\[-4 + (-12) + 4 = -4 + 4 + (-12) \quad \text{Commutative Property}\]
\[= 0 + (-12) \quad \text{Additive Inverse Property}\]
\[= -12 \quad \text{Identity Property of Addition}\]

Example 2.13

The starting balance in a bank account is 500 birr. What is the balance after 120 Birr, and 200 Birr are withdrawn from the account?

Solution:

Withdrawing decreases your account balance, the withdrawals can be represented by the integers -120 and -200. Add these integers to the starting balance to find the new balance.

\[500 + (-120) + (-200) = 500 + [-120 + (-200)] \quad \text{Associative Property}\]
\[= 500 + (-320) \quad -120 + (-200) = -320\]
\[= 180\]

Therefore, the balance is now 180 birr.

Example 2.14

Compute the following using commutative property of addition.

a. \(-15 + (-13) + (-5)\)

b. \(18 + 25 - 8\)
Solution:

a. \(-15 + (-13) + (-5) = -15 + (-5) + (-13)\) . Why?
   \[= (-15 + (-5)) + (-13)\] . Why?
   \[= (-20) + (-13)\]
   \[= -33\]

b. \(18 + 25 - 8 = 18 + 25 + (-8)\)
   \[= 18 + (-8) + 25\] why?
   \[= 10 + 25 = 35\]

Exercise 2.7

1. Simplify the following expressions, if necessary use properties.
   a. \(53 + 18 + 47\)
   d. \(-23 + (-56) + 23\)
   b. \(-15 + (-13) + (-5)\)
   e. \((144 + 40) + (-40)\)
   c. \(125 +((-25) + 15)\)
   f. \(-12 + 19 + (-14) + 1\)

2. There were 2785 students in a school. At the beginning of the year 42 students left the school due to different reasons and 250 new students were admitted. At present how many students are there at the school?

3. The highest temperature of a town was \(-8^\circ F\) on Monday and \(11^\circ F\) on Tuesday. How much warmer was it on Tuesday?

4. A bank account has a starting balance of 130 Birr. What is the balance after withdrawing 58 Birr and 62 Birr, and then making a deposit of 150 Birr?

5. Is the sum of any two integers always an integer?

2.4.2 Subtraction of Integers

Activity 2.8

You can use number lines to model subtraction problems.
UNIT 2: INTEGERS

1. Use a number line to model \(-1 + (-4)\).

2. Compare this model to the model for \(-1 - 4\). How is \(-1 - 4\) related to \(-1 + (-4)\)?

The activity shows that when you subtract a number, the result is the same as adding the opposite of the number.

When you subtract 5, the result is the same as adding its opposite, \(-5\).
When you subtract 4, the result is the same as adding its opposite, \(-4\).

\[
\begin{align*}
3 - 5 &= -2 & 3 + (-5) &= -2 \\
-1 - 4 &= -5 & -1 + (-4) &= -5
\end{align*}
\]

To subtract integers, you can use counters or the following rule.

**Remark**

If \(a\) and \(b\) are any two integers, then, \(a - b = a + (-b)\).

Therefore, to subtract an integer, add it’s opposite.

**Example 2.15**

Find: \(8 - 11\)

**Solution:**

**Method: 1 Using counters**
First place 8 positive counters to show +8.
However, there are no enough counters to take 11 positive counters away.
So, add 3 zero pairs to get 11 positive counters.

Now, remove 11 positive counters. This leaves 3 negative counters.
Therefore, \( 8 - 11 = -3 \)

**Method 2  Adding opposites**

\[
8 - 11 = 8 + (-11) \quad \text{To subtract 11, add } -11.
\]

\[= -3\]

**Example 2.16**

Find: \(-12 - 8\)

**Solution:**

\[
-12 - 8 = -12 + (-8) \quad \text{To subtract 8, add } -8.
\]

\[= -20\]

**Example 2.17**

Find:

a. \(7 - (-3)\)  

b. \(-9 - (-4)\)

**Solution:**

a. \(7 - (-3) = 7 + 3 \) \(\text{To subtract } -3, \text{ add } 3.\)

\[= 10\]

b. \(-9 - (-4) = -9 + 4 \) \(\text{To subtract } -4, \text{ add } 4.\)

\[= -5\]

**Example 2.18**

The minimum and maximum temperatures of Debre Birhan city in the month of December is recorded as \(-5^\circ C\) and \(22^\circ C\). What is the difference between the maximum and the minimum temperatures?
UNIT 2: INTEGERS

Solution:

To find the difference in temperatures, subtract the lower temperature from the higher temperature.

\[ 22 - (-5) = 22 + 5 \text{ To subtract -5, add 5.} \]

\[ = 27. \]

So, the difference between the temperatures is 27°C.

---

Exercise 2.8

1. Subtract each of the following. Use counters or number line or adding opposites, if necessary.
   a. \( 6 - 12 \)
   b. \( -20 - 15 \)
   c. \( -22 - 26 \)
   d. \( 4 - (-12) \)
   e. \( -15 - (-5) \)
   f. \( 18 - (-6) \)
   g. \( -3 - (-1) \)
   h. \( 10 - 30 \)

2. Subtract.
   a. \( -11 - (-9) \)
   b. \( -2 - 23 \)
   c. \( 14 - (-10) \)
   d. \( 5 - (-16) \)
   e. \( 52 - (-52) \)
   f. \( 15 - (-14) \)
   g. \( -27 - (-33) \)
   h. \( -18 - (-20) \)

3. The temperatures on the moon vary from -173°C to 127°C. Find the difference in temperatures.

4. The Dallol depression in Afar region is 116 meters below sea level. The Ras Dejen (also known as Ras Dashen) mountain in Amhara region rises to about 4,550 meters above sea level. What is the difference between the highest part of Ras Dejen and the deepest part of Dallol?

Problem Solving

1. Keeping the single-digit numbers from 1 to 9 in order, 1 2 3 4 5 6 7 8 9 and inserting plus and/or minus signs, you can obtain a sum of 100 in several ways. For example,

\[ 1 + 2 + 3 - 4 + 5 + 6 + 7 + 8 + 9 = 100. \]

Find at least three different ways.

2. In one hour, an elevator traveled up 5 floors, down 12 floors, up 8 floors, down 6 floors, up 11 floors, and down 12 floors. If the elevator started on 7th floor, on which floor is it now?
2.5 Multiplication and Division of Integers

2.5.1 Multiplication of Integers

Activity 2.9

Just as $4 \times 2$ means 4 groups of 2, $4 \times (-2)$ means 4 groups of (-2). Place 4 sets of 2 negative counters.

1. Write a multiplication sentence that describes the model above.
2. Use counters to find $3 \times (-2)$, $5 \times (-2)$ and $4 \times (-3)$.
3. What is the sign of the product of a positive and a negative integer?

Remember that multiplication is the same as repeated addition. The multiplication expression $4 \times (-2)$ in the activity means that -2 is used as an addend four times.

By the Commutative Property of Multiplication, $4 \times (-2) = -2 \times 4$. When two integers have different signs, the following rule applies.

Note: The product of two integers with different signs is negative.

For example, $3 \times (-6) = -18$ and $-6 \times 3 = -18$

Example 2.19

Find:

a. $3 \times (-2)$  

b. $-12 \times (3)$

Solution:

a. $3 \times (-2)$ means 3 groups of (-2). Place 3 sets of 2 negative counters together. There are a total of 6 negative counters. Therefore, $3 \times (-2) = -6$. 

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b. \(-12 \times (3) = -(12 \times 3) = -36\). The integers have different signs. So, the product is negative.

The product of two positive integers is positive. What is the sign of the product of two negative integers? Look at the pattern below.

\[
\begin{align*}
3 \times 4 &= 12 \\
2 \times 4 &= 8 \\
1 \times 4 &= 4 \\
0 \times 4 &= 0 \\
-1 \times 4 &= -4 \\
-2 \times 4 &= -8 \\
-3 \times 4 &= ?
\end{align*}
\]

\[
\begin{align*}
3 \times (-4) &= -12 \\
2 \times (-4) &= -8 \\
1 \times (-4) &= -4 \\
0 \times (-4) &= 0 \\
-1 \times (-4) &= 4 \\
-2 \times (-4) &= 8 \\
-3 \times (-4) &= ?
\end{align*}
\]

By extending the number pattern, you find that
- \(-3 \times 4 = -12\)
- \(-3 \times (-4) = 12\)

**Note:**

The product of two integers with the same sign is positive.

For example, \(-4 \times (-7) = 28\) and \(-7 \times (-4) = 28\)

**Example 2.20**

Find: \(-5 \times (-6)\)

**Solution:**

\(-5 \times (-6) = 30\). The integers have the same sign. The product is positive.

**Note:**

To find the product of two integers,

1. take the integers as positive and find the product,
2. put the sign in front of the product.

**Example 2.21**

Find the products

a. \(20 \times 3\)  
   b. \(-15 \times 5\)  
   c. \(-30 \times -4\)
Solution:

a. The factors 20 and 3 are positive, then, their product is also positive. Therefore, \(20 \times 3 = 60\).

b. The factors -15 and 5 have opposite signs, then, their product is negative. Therefore, \(-15 \times 5 = -75\).

c. The factors -30 and -4 are negative, then, their product is positive. Therefore, \(-30 \times (-4) = 120\).

Exercise 2.9

1. Multiply. Use counters or number line, if necessary.
   a. 9 (-2)  
   d. 5(-3)  
   g. 6(12) 
   b. -7 (4)  
   e. -8(6)  
   h. 1,234,567×(-1) 
   c. -3(-7)  
   f. -16(-5)

2. For each kilometer above the Earth’s surface, the temperature decreases by 7°C. If the temperature at the Earth’s surface is 0°C, then what will the temperature be on 2 kilometers above the surface?

3. On Wednesday morning, the temperature of Bahir Dar city dropped 4°C every hour for 5 hours. If the temperature was 21°C before it started dropping, then what was the temperature after 5 hours?

Activity 2.10

1. Check whether the following pairs of products are equal or not.
   a. 3×4 and 4×3 
   b. 5×(-2) and -2×5 
   c. -2×(5×3) and (-2×5)×3 
   d. -2×(5+3) and (-2×5)+(-2×3) 
   e. 3×(2-4) and (3×2)-(3×4) 

2. What do you understand from the above paired values?

From the above activity, you can understand that, when you multiply two or more integers, their product is the same regardless of the order or the grouping of the factors. Based on your work on the activity, the following properties of multiplication on integers can be generalized. These properties can help you multiply three or more integers.
UNIT 2: INTEGERS

Properties of Multiplication on Integers

a. Commutative property of multiplication.
   For any two integers a and b, \( a \times b = b \times a \).
   For example, \(-4 \times 6 = 6 \times (-4) = -24\).

b. Associative property of multiplication.
   For any three integers a, b, and c; \( (a \times b) \times c = a \times (b \times c) \).
   For example, \((-5 \times 4)\times 2 = -5 \times (4 \times 2) = -40\).

c. Distributive property of multiplication over addition.
   For any three integers a, b, and c, \( a \times (b+c) = (a \times b) + (a \times c) \).
   For example, \(-3 \times (2 + (-3)) = (-3 \times 2) + ((-3) \times (-3)) = 3\)

d. Distributive property of multiplication over subtraction.
   For any three integers a, b, and c, \( a \times (b - c) = (a \times b) - (a \times c) \).

e. Property of 1 for multiplication.
   For any integer \( a; a \times 1 = 1 \times a = a \)
   For example, \(-4 \times 1 = 1 \times (-4) = -4\).
   1 is called the identity element for multiplication on integers.

Example 2.22

Find \(-20 \times 16 \times 5\).

Solution:

\[-20 \times 16 \times 5 = -20 \times 5 \times 16 \text{ Why?} \]
\[-100 \times 16 = -1600.\]

Example 2.23

Find: \(-2 \times 12 \times (-5)\)

Solution:

Method: 1  Use the Associative Property.

\[-2 \times 12 \times (-5) = [-2 \times 12] \times (-5) \text{ Associative Property} \]
\[= -24 \times (-5), \text{ because } -2 \times 12 = -24 \]
\[= 120, \text{ because } -24 \times (-5) = 120\]
Method: 2 Use the Commutative Property.

\[-2(12)(-5) = -2(-5)(12) \quad \text{Commutative Property} \]
\[= 10(12), \text{ because } -2(-5) = 10 \]
\[= 120, \text{ because } 10(12) = 120 \]

Example 2.24
Evaluate \(xyz\) if \(x = -3, y = -4,\) and \(z = -2.\)

Solution:
\[xyz = -3 \times (-4) \times (-2), \quad \text{Replace } x \text{ with } -3, y \text{ with } -4, \text{ and } z \text{ with } -2. \]
\[= [-3 \times (-4)] \times (-2) \quad \text{Associative Property} \]
\[= 12 \times (-2) \]
\[= -24 \]

Note:
- If the number of negative integers to be multiplied is odd, the product will be negative. 
  Example: \(-3 \times (-4) \times (-3) = -24\)
- If the number of negative integers to be multiplied is even, the product will be positive. 
  Example: \(-4 \times (-3) \times (-2) \times (-5) = 120\)

Exercise 2.10

1. Check whether the following statements are true or false.
   a. \(-120 \times 35 = 35 \times (-120)\)
   b. \(34 \times (-10 \times 25) = (34 \times (-10)) \times 25\)
   c. \(12 \times (24 + (-12)) = (12 \times 24) + (-12)\)

2. Multiply.
   a. \(4(-2)(-5)\)
   b. \(-1(-3)(8)\)
   c. \(-2 \times (-5) \times (-1)\)
   d. \(3(-3)(4)\)
   e. \(-8 \times 4 \times (-5)\)
   f. \(-4 \times (-2)(-8)\)
   g. \(9 (-1)(-5)\)
   h. \(2 (4) (5)\)

3. Evaluate \(pqr\) if \(p = -7, q = -4,\) and \(r = 2.\)
UNIT 2: INTEGERS

4. If the temperature of a town now reads 0°C, then what will be the temperature
   a. 4 hours later if it increases 3°C each hour?
   b. 5 hours later if it decreases 2°C each hour?
   c. 3 hours ago, if it has been increasing 5°C each hour?
   d. 4 hours ago, if it has been decreasing 2°C each hour?

5. A carton holds 24 packets of biscuits. Each packet has 12 biscuits. How many
   biscuits can be packed in 45 cartons?

6. For any two integers $a$ and $b$, $a \times b$ is also an integer. Is this statement true? Why?

2.5.2 Division of Integers

Activity 2.11

You can use counters to model division of integers. Follow these steps to find $-12 \div 3$.

Step 1: Place 12 negative counters in the box to represent $-12$.

Step 2: Arrange the 12 negative counters into 3 equal-size groups.
   There are 4 negative counters in each group.
   So, $-12 \div 3 = -4$.

Find each quotient using counters or a drawing.

a. $-9 \div 3$
   b. $-10 \div 2$

Division of numbers is related to multiplication. When finding the quotient of two integers, you can use a related multiplication sentence.

Factors

3 \( \times \) 5 = 15
4 \( \times \) 6 = 24

Quotients

15 \( \div \) 3 = 5
24 \( \div \) 4 = 6

Note that: In the division sentence, 15 and 24 are dividends; 3 and 4 are divisors; and 5 and 6 are quotients.

Since multiplication and division sentences are related, you can use them to find the quotient of integers with different signs.

\[ 3 \times (-5) = -15 \quad -15 \div 3 = -5 \]
\[ -3 \times (-5) = 15 \quad 15 \div (-3) = -5 \]
These examples suggest that the rules for dividing integers are similar to the rules for multiplying integers.

**Rules of Division of Integers**

a. If the dividend and divisor have the same sign, then the quotient is positive.

b. If the dividend and divisor have different signs, then the quotient is negative.

For example, 16 ÷ (-8) = -2 , and -16 ÷ (-8) = 2.

c. The rules for division of integers are same as multiplication rules.

---

**Example 2.25**

Express the following division equations by the equivalent multiplication equation.

a. -15÷3 = - 5

b. 75÷(-15) = 5

**Solution:**

a. -15÷3 = - 5, because -5×3 = -15.

b. -75÷(-15) = 5, because 5×(-15)=-75.

---

**Example 2.26**

Find:

a. -36 ÷ 4.

b. -20 ÷ ( -4)

**Solution:**

a. -36 ÷ 4 = -9. The dividend and the divisor have different signs.

Hence, the quotient is negative.

b. -20 ÷ (-4) = 5. The dividend and the divisor have the same sign.

Hence, the quotient is positive.

You can use all of the rules you have learned for adding, subtracting, multiplying, and dividing integers to evaluate expressions.

---

**Example 2.27**

Evaluate -3x – y, if x = -2 and y = -7.
Solution:

\[-3x - y = -3(2) - (-7)\]

Replace \(x\) with -2 and \(y\) with -7.

\[= 6 - (-7)\]

The product of -3 and -2 is positive.

\[= 6 + 7\]

To subtract -7, add 7.

\[= 13\]

Example 2.28

On six consecutive days, the lowest temperature in a certain town was -6°C, -5°C, 6°C, 3°C, -2°C, and -8°C. What was the average of the lowest temperature for the six days?

Solution:

Average Temperature \(= \frac{-6 + (-5) + 6 + 3 + (-2) + (-8)}{6}\)

\[= \frac{-11 + 9 + (-10)}{6}\]

\[= \frac{-12}{6} = -2\]

Hence, the average of the lowest temperature is -2°C.

Exercise 2.11

1. Complete the following table by changing division sentence into multiplication sentence and vice versa.

<table>
<thead>
<tr>
<th>Division sentence</th>
<th>Multiplication Sentence</th>
</tr>
</thead>
<tbody>
<tr>
<td>15 ÷ 5 = 3</td>
<td>35 = -7 × (-5)</td>
</tr>
<tr>
<td>-40 ÷ 5 = -8</td>
<td>-30 = -6 × 5</td>
</tr>
<tr>
<td>14 ÷ (-2) = -7</td>
<td></td>
</tr>
</tbody>
</table>

2. Divide.

a. \((-80)÷20\)
   
e. \(\frac{28}{7}\)
   
g. \(\frac{-96}{12}\)

b. \(40÷(-5)\)
   
c. \((-150)÷(-5)\)
   
f. \(\frac{-25}{-5}\)
   
h. \(\frac{-72}{-4}\)

d. \(120÷(-3)\)
3. Are the quotients in question 1 above all integers? If your answer is no, identify which quotients are not integers. What can you conclude in general?

4. Evaluate each expression if \( x = -5 \), \( y = 7 \), and \( z = -13 \).
   
   a. \( 5x + 9 \)  
   b. \( -3y - 1 \)  
   c. \( \frac{y - z}{x} \)

5. When are the following true?
   
   a. \( a \div b = 0 \)  
   b. \( a \div b = 1 \)  
   c. \( a \div b = a \)

6. Most people lose 100 to 200 hairs per day. If you were to lose 150 hairs per day for 10 days, what would be the number of hairs you lose?

### 2.6 Even and Odd Integers

**Activity 2.12**

1. List all natural numbers less than 15, which are divisible by 2. Are the opposites of these numbers divisible by 2?

2. List all natural numbers less than 15, which are not divisible by 2. Do you think that the opposites of these numbers are not also divisible by 2?

3. Is 0 divisible by 2 or not? Give your reason.

From your responses on the activity, it is evident that natural numbers which are divisible by 2 have opposites which are also divisible by 2. These natural numbers together with their opposites are said to be even integers.

Similarly, natural numbers which are not divisible by 2 have opposites which are also not divisible by 2. These numbers are said to be odd integers.

You can list odd and even integers as below.

- Odd integers: \( \{..., -5, -3, -1, 1, 3, 5, 7, ...\} \)
- Even integers: \( \{..., -6, -4, -2, 0, 2, 4, 6, ...\} \)

**Example 2.29**

Show that for any integer, \( k \), even integers can be written in the form of \( 2k \), and odd integers can also be written as \( 2k + 1 \).
Solution:
Let us take few integers as examples, and look at the pattern to generalize.

\[
\begin{align*}
-5 &= 2 \times (-3) + 1 \quad \text{odd} & \quad -4 &= 2 \times (-2) \quad \text{even} \\
-3 &= 2 \times (-2) + 1 \quad \text{odd} & \quad -2 &= 2 \times (-1) \quad \text{even} \\
-1 &= 2 \times (-1) + 1 \quad \text{odd} & \quad 0 &= 2 \times 0 \quad \text{even} \\
1 &= 2 \times 0 + 1 \quad \text{odd} & \quad 2 &= 2 \times 1 \quad \text{even} \\
3 &= 2 \times 1 + 1 \quad \text{odd} & \quad 4 &= 2 \times 2 \quad \text{even} \\
5 &= 2 \times 2 + 1 \quad \text{odd} & \quad 6 &= 2 \times 3 \quad \text{even}
\end{align*}
\]

As you can see from the pattern, for any integer \( k \), we can write odd integers as \( 2k + 1 \), and even integers as \( 2k \).

Activity 2.13

1. Take any two even integers, and then find their sum. Is the sum even or odd?
   
   What if the addends are three even integers? Four even integers?
   
   What do you conclude from this?

2. Take any two even integers, and then find their product. Is the product odd or even?
   
   What if the factors are three even integers? Four even integers?
   
   What do you conclude from this?

From the above activity, you can conclude that the sum or product of two or more even integers is also even integer.

Can we conclude that the sum and product of two or more odd integers is also odd? How about the sum and product of odd and even integers? To answer these questions, look at the following examples.

Example 2.30

Compute the following and check whether their sum or product is odd or even.

\begin{align*}
a. \quad 15+91 & \quad \text{c.} \quad 201+(-21) \\
b. \quad -25+45 & \quad \text{d.} \quad -35+(-13)
\end{align*}

Solution:

\[
\begin{array}{|c|c|c|c|}
\hline
\text{a.} & 15+91=106 & \text{b.} & -47+25=-22 \\
\quad & \text{even} & \quad & \text{even} \\
\text{c.} & 201+(-21)=180 & \text{d.} & -35+(-13)=-48 \\
\quad & \text{even} & \quad & \text{even} \\
\hline
\end{array}
\]
Example 2.31

a. 11×7  
   odd

b. -23×3  
   odd

c. 19×(-5)  
   odd

d. -27×(-3)  
   odd

Solution:

<table>
<thead>
<tr>
<th></th>
<th>a. 11×7=77</th>
<th>b. -23×3=69</th>
<th>c. 19×(-5)=95</th>
<th>d. -27×(-3)=81</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>odd</td>
<td>odd</td>
<td>odd</td>
<td>odd</td>
</tr>
</tbody>
</table>

From the two examples above, you have observed that the sum of two odd integers is even, and the product of two odd integers is odd. You can also check the result for any other pairs of odd integers.

Example 2.32

Compute the following and check whether their sum or product is odd or even.

a. 35 + 41 + (-13)  
   d. -15×(-7)×11×(-9)

b. 102 + (-204) + 78  
   e. 45 + (-64) + 15

c. -12 × 4 × (-8)  
   f. -5×18×23

Solution:

a. 35 + 41 + (-13) = 76 + (-13) = 63 (Since all the three addends are odd their sum is odd)

b. 102 + (-204) + 78 = -102 + 78 = -24 (Since all the three addends are even their sum is even)

c. -12×4×(-8)=1344 (Since all the factors are even their product is even)

d. -15×(-7)×11×(-9)=10395 (Since all the factors are odd their product is odd)

e. 45 + (-64) + 15 = -4 (Since the number of odd addends are even their sum is even, and the sum of evens is even)

f. -5×18×23=-2070 (Since there is one even factor their product is even)

Exercise 2.12

1. Fill the following table by writing the word even or odd

<table>
<thead>
<tr>
<th></th>
<th>Even</th>
<th>Odd</th>
</tr>
</thead>
<tbody>
<tr>
<td>a.</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>even</th>
<th>Odd</th>
</tr>
</thead>
<tbody>
<tr>
<td>b.</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Even</th>
<th>Odd</th>
</tr>
</thead>
<tbody>
<tr>
<td>c.</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Even</th>
<th>Odd</th>
</tr>
</thead>
<tbody>
<tr>
<td>Even</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Odd</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
UNIT 2: INT(Integer)s

2. Identify whether the results of each of the following sum or product of integers is odd or even.
   a. \(-35 + 45 + (-71)\)  
ed. \(-35 + 45 + (-71)\)
   b. \(-208 + (-34) + 86 + 94\)  
e. \(84 + 75 + (-59)\)
   c. \(-26 \times 56 \times (-6)\)  
f. \(-12 \times 17 \times (-1001)\)
   d. \(-7 \times (-19) \times 21\)  
g. odd + odd + odd + even
   h. even + odd + even + even + odd

3. How many odd integers can be added to get odd sum? Even sum?
4. How many odd and even integers can be added to get odd sum? Even sum?
5. Show that for any integer \(k\), odd integers can also be written as \(2k-1\).

Unit Review

1. For any three integers \(a\), \(b\) and \(c\)
   
   \[a + b = b + a\]  
   Commutative property of addition
   
   \[a \times b = b \times a\]  
   Commutative property of multiplication
   
   \[a + (b + c) = (a + b) + c\]  
   Associative property of addition
   
   \[a \times (b \times c) = (a \times b) \times c\]  
   Associative property of multiplication
   
   \[a \times (b \pm c) = (a \times b) \pm (a \times c)\]  
   Distributive property of multiplication over addition (subtraction)
   
   \[a + 0 = 0 + a = a\]  
   Additive identity
   
   \[a + (-a) = (-a) + a = 0\]  
   Additive inverse
   
   \[a \times 1 = 1 \times a = a\]  
   Multiplicative identity
   
   \[a \times \frac{1}{a} = \frac{1}{a} \times a = 1, \quad a \neq 0\]  
   Multiplicative inverse
   
   \[\text{Opposite of opposite}\]

\(\Rightarrow\) Rules of signs for addition

\(\Rightarrow\) Sum of a positive and a negative integer is positive if one of the integers is greater than opposite of the other integer.

\(\Rightarrow\) Sum of a positive and a negative integer is negative if one of the integers is less than opposite of the other integer.

\(\Rightarrow\) Rules of signs for multiplication

\(\Rightarrow\) Product of two integers having the same signs is positive

\(\Rightarrow\) Product of two integers having different signs is negative
Rules of signs for division

- Quotients of integers having the same signs are positive
- Quotients of integers having different signs are negative

The sum of n odd integers is even if n is even, and odd if n is odd.

The set of integers is closed under addition, subtraction and multiplication, but not in division.

On the set of integers addition and multiplication are commutative and associative, but, subtraction and division are not.

## Review Exercises

1. Draw a number line and represent each of the following on the line?
   
   a. \(-5\) + \((-7)\)  
   b. \(4\) + \((-12)\) + \(9\)  
   c. \(-6\times3\) + \((-10)\)  
   d. \(3\times(-8)\)

2. Compare the following pairs of integers
   
   a. \(-999\) ________ \(-1001\)  
   b. \(-6\times(-9)\) ________ \(-7\times(-8)\)  
   c. \(-12-12\) ________ \(-12+(-12)\)  
   d. \(15\times(13\div5)\) ________ \(13\times(15\div5)\)

3. Arrange the following integers in descending order
   
   \(-12, -21, -23, -25, 27, -72, -101, 66, -99, -1, -13, 52\)

4. Simplify each of the following
   
   a. \(72 \div (18 - 6) + 13 \times 2 + 8 - 3\)  
   b. \(-20 \times (18 - (-18))\)  
   c. \(-16 + 32 - (-8) \times 5\)  
   d. \(-45 \times 17 \times (-26)\)  
   e. \(-300 + 908 - 609 \times (-18)\)  
   f. \(-8 \times (-3) \times (-1)\)

5. Evaluate
   
   a. \(a - b\), if \(a = -4\) and \(b = 10\)
   b. Evaluate \(b - a\) for the same values of \(a = -4\) and \(b = 10\)
   c. From your results how do you relate \(a - b\) and \(b - a\)?
   d. Does your conclusion hold true if \(a = 7\) and \(b = 9\)?
6. Find the additive inverse (opposite) and multiplicative inverse (reciprocals) of each of the following integers if it exists
   a. 1
   b. 0
   c. -1
   d. -2

7. For any three integers a, b and c such that c ≠ 0, identify which of the following are true? Explain
   a. \( a + (b - c) = (a + b) - (a + c) \)
   b. \( (a + b) \div c = (a \div c) + (b \div c) \)
   c. \( a - (b - c) = (a - b) - c \)
   d. \( a \times (b \times c) = (a \times b) \times c \)

8. If \( 12 \times (5 + 8) = (12 \times y) + (12 \times 8) \), then what is the value of y?

9. Take any two natural numbers a and b. Find their GCF and LCM and that \( \text{GCF}(a, b) = \frac{a \times b}{\text{LCM}(a, b)} \) is true.

10. In the triangular shape shown at the right, place the whole numbers 1 through 9 in the circles so that the sum of the numbers in each side of the triangle is 17.

11. Fill in each empty square so that the number in the square will be the sum of the pairs of numbers beneath the square.

12. Place the numbers -6, -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5, 6, 7 one in each of the regions of the seven circles below, so that the sum of the three numbers in each circle is zero.
UNIT - 3
RATIO, PROPORTION AND PERCENTAGE

Learning Outcomes:

After completing this unit, you will be able to:

✎ Solve problems involving ratio and proportion.
✎ Describe a percentage.
✎ Solve problems involving percentages.
✎ Relate fractions, decimals and percentages to real life situations
✎ Apply the concept of percentage in solving real life problems.

Key Terms

✎ Ratio
✎ Proportion
✎ Percentage
✎ Profit, Loss
✎ Percentage profit
✎ Percentage loss
✎ Simple interest
✎ Compound interest
✎ Tax – income tax, Value added tax (VAT)
✎ Turn over tax (TOT)
UNIT 3: RATIO, PROPORTION AND PERCENTAGE

Introduction

In a real situation people are comparing two or more quantities that are measured in the same unit. Can you give examples of such quantities? Have you ever compared such quantities by yourself or with your friends? In this unit, you will learn mathematical concept of comparing quantities such as ratio, proportion and percentage. You will also see the application of percentage to calculate profit, loss, interest and different types of Ethiopian Taxes.

3.1 Ratio and Proportion

3.1.1 Ratio

Activity 3.1

1. Write a simple comparison for each of the following.
   a. Number of blue cups to red cups.
   b. Number of triangles to rectangles.
   c. Amounts of coffee to milk.

2. What does such comparison tell us?

There are many different ways to compare amounts or quantities. A ratio is a comparison of two quantities by division. A ratio of 3 blue cups to 6 red cups can be written in three ways.

<table>
<thead>
<tr>
<th>Ratio</th>
<th>Using ‘to’</th>
<th>Using ‘:’ sign</th>
<th>Using as a fraction</th>
</tr>
</thead>
<tbody>
<tr>
<td>blue cups to red cups</td>
<td>3 to 6</td>
<td>3 : 6</td>
<td>( \frac{3}{6} )</td>
</tr>
</tbody>
</table>

Read each ratio as three to six.

As with fractions, ratios are often expressed in simplest form.
Example 3.1

Write the ratio that compares the number of blue cups to the number of red cups in simplest form. Then explain its meaning.

Solution:

Since the GCF of 3 and 6 is 3, we have:

\[
\frac{3 \div 3}{6 \div 3} = \frac{1}{2}
\]

The ratio of blue to red cups is: \( \frac{1}{2} \), 1 to 2, or 1 : 2.

This means that for every 1 blue cup there are 2 red cups.

Example 3.2

Write the ratio of the first number to the second one in simplest form.

\( a. \) 40 and 200  \hspace{1cm} \( b. \) 240 and 90  \hspace{1cm} \( c. \) 48 and 108

Solution:

\( a. \) The ratio 40 to 200 is \[
\frac{40}{200} = \frac{1 \times 40}{5 \times 40} = \frac{1}{5}
\]

\( b. \) The ratio 240 to 90 is \[
\frac{240}{90} = \frac{8 \times 30}{3 \times 30} = \frac{8}{3}
\]

\( c. \) The ratio 48 to 108 is \[
\frac{48}{108} = \frac{4 \times 12}{9 \times 12} = \frac{4}{9}
\]

Example 3.3

A class has 22 boys and 18 girls. What is the ratio of boys to girls? Write in simplest form.

Solution:

The ratio of boys to girls is: \[
\frac{22}{18} = \frac{11 \times 2}{9 \times 2} = \frac{11}{9}
\]

Therefore, the ratio of boys to girls in simplest form is 11to 9 or 11:9

Note: When comparing two or more quantities in terms of ratio, one must bear in mind that:

a. The quantities must be of the same kind and their units of measurement must be identical.

b. For two quantities \( a \neq b \), \( a:b \neq b:a \)

c. Ratio has no unit (it is simply a number).
UNIT 3: RATIO, PROPORTION AND PERCENTAGE

d. In most cases, the ratio a:b is written in simplified form where a and b are natural numbers.

Exercise 3.1

1. Write the ratio of the first number to the second one in simplest form.
   a. 48 and 80
   b. 4.8 and 9.6
   c. \( \frac{14}{21} \) and \( \frac{10}{15} \)
   d. 0.54 and 0.09

2. Write each ratio in simplest form.
   a. teachers : students
   b. classrooms : people
   c. students : classrooms
   d. teachers : people

   | Grade 7 Students Statistics                     |
   | Students | 180       |
   | Teachers  | 12        |
   | Classrooms | 4        |

3. Express each of the following ratios as fractions in their lowest terms.
   a. 4 Birr to 16 cents
   b. 5 days to 100 hours
   c. 3.5 kilograms to 6500 grams
   d. 2 kilometers to 2250 meters
   e. 3 minutes 54 seconds to 2 minutes 6 seconds

4. A theater is showing 8 comedies and 12 action movies. What is the ratio of action movies to comedies? Write in simplest form.

Activity 3.2

Suppose in your class, there are 24 girls and 16 boys.

a. What is the ratio of girls to boys?

b. What is the ratio of boys to girls?

c. What is the ratio of girls to the total number of students?

d. What is the ratio of boys to the total number of students?

Ratios can be used to compare a part to a part, or a part to a whole.

Part-to-part Ratio: A ratio that can relate one part of a whole quantity to another part of the quantity. A part-to-part ratio does not reflect the concept of fraction. Although it can be written with the fraction bar, the context tells you it is a part-to-part ratio. For example, the ratio of number of:

- boys to girls in a classroom,
- mathematics teachers to physics teachers,
- Population of one region to another region, etc.
**Part-to-whole ratio:** A ratio that can express comparison of a part to a whole quantity. This can be written as fraction. Moreover, part – to – whole ratio reflects the concept of fraction. For example, the ratio of the number of:

- boys in a class to total number of students in the class,
- mathematics teachers in the school to total number of teachers in the school,
- population of Amhara region to total population of Ethiopia, etc.

**Example 3.4**

Last month, Aster ate 9 avocados, 5 bananas, 4 peaches, and 7 oranges.

Find the ratio, in simplest form, of bananas to the total number of pieces of fruit Aster ate last month.

**Solution:**

The ratio of banana to the whole fruits is: \[
\frac{5}{9 + 5 + 4 + 7} = \frac{5}{25} = \frac{1}{5}
\]

**Example 3.5**

Share 400 in the ratio of 1:3?

**Solution:**

The sum of the parts is 1+3=4. Then,

- The first part is \( \frac{1}{4} \) of 400 = \( \frac{1}{4} \times 400 = 100 \)
- The second part is \( \frac{3}{4} \) of 400 = \( \frac{3}{4} \times 400 = 300 \)

Therefore, when 400 is divided in the ratio, the first part is 100 and the second part is 300.

**Note:** For quantity Q and ratio a : b, we can generalize using the formulas:

First part = \( \left( \frac{a}{a+b} \right)Q \) and second part = \( \left( \frac{b}{a+b} \right)Q \)

**Example 3.6**

An artist made 36 gallons of paint using white pigment, kerosene, and dryer in the ratio of 3:2:1 respectively. How many gallons of each material did the artist used? What do you say about the sum of the amount of each material?

**Solution:**

The sum of the parts Q is 3+2+1=6, then

- The white pigment is \( \frac{3}{6} \) of 36 gallons = \( \frac{3}{6} \times 36 \text{ gallons} = 18 \text{ gallons} \)
UNIT 3: RATIO, PROPORTION AND PERCENTAGE

- The kerosene is \( \frac{2}{6} \) of 36 gallons = \( \frac{2}{6} \times 36 \) gallons = 12 gallons

- The dryer is \( \frac{1}{6} \) of 36 gallons = \( \frac{1}{6} \times 36 \) gallons = 6 gallons

Therefore, the artist used 18 gallons white pigment, 12 gallons kerosene and 6 gallons dryer to make the paint.

Note: For ratios with three terms \( a : b : c \) and quantity \( Q \), we can write each part as:

First part = \( \left( \frac{a}{a+b+c} \right) Q \), second part = \( \left( \frac{b}{a+b+c} \right) Q \) and third part = \( \left( \frac{c}{a+b+c} \right) Q \)

Example 3.7

If \( a, b \) and \( c \) are numbers such that \( a:b:c = 3:4:5 \) and \( b=20 \). Find the sum of \( a+b+c \).

Solution:

Since the ratio of numbers \( a, b \) and \( c \) is \( a:b:c = 3:4:5 \), we have

\[
\frac{a}{b} = \frac{3}{4}, \quad \frac{b}{c} = \frac{4}{5}, \quad \frac{a}{c} = \frac{3}{5}
\]

Then, since \( b = 20 \), from the first ratio \( \frac{a}{b} = \frac{3}{4} \)

We have \( a = \frac{3}{4} \times 20 = \frac{3}{4} \times 20 = 15 \)

Similarly, from the last ratios, we have \( \frac{b}{c} = \frac{20}{c} = \frac{4}{5} \). Then since \( b = 20 \), from the first equation.

We obtain, which implies that \( 4c=20\times5=100 \), which implies that \( c=25 \).

Therefore, \( a+b+c = 15+20+25 = 60 \).

Example 3.8

Find the ratio of \( a \) to \( b \), if \( 2a = 3c \) and \( 12c = 7b \)

Solution:

Solving for \( a \) and \( b \) gives \( a = \frac{3}{2} c \), \( b = \frac{12}{7} c \)

Then the ratio is given by \( \frac{3c}{2} = \frac{2}{3c} \times \frac{7}{2c} = \frac{7}{8} \)

Therefore, the ratio of \( a \) to \( b \) is 7:8.
Exercise 3.2

1. If a rural veterinarian treated only 12 cows and 16 horses last week, then
   a. What is the ratio of treated cows to horses?
   b. What is the ratio of treated cows to the total animals in that week?

2. Use the following information about university students to complete the following exercises

<table>
<thead>
<tr>
<th>Gender</th>
<th>Band</th>
<th>Batch</th>
</tr>
</thead>
<tbody>
<tr>
<td>Boys</td>
<td>Natural science 160</td>
<td>Freshmen 127</td>
</tr>
<tr>
<td>Girls</td>
<td>Social science 56</td>
<td>2nd year 63</td>
</tr>
<tr>
<td></td>
<td>Engineering 84</td>
<td>3rd year 55</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Seniors 55</td>
</tr>
</tbody>
</table>

   a. What is the ratio of boys to girls?
   b. What is the ratio of freshmen to the total number of students?
   c. What is the ratio of engineering to natural science students?
   d. What is the ratio of freshmen to 2nd year students?
   e. What is the ratio of natural science to engineering to social science students?

3. Find the two numbers
   a. whose ratio is 3:5 and their sum is 80.
   b. whose ratio is 3:7, and their difference is 20.

4. If 2A:3B = 5:6 and 3B:2C = 36:15, then find A:C.

5. A sugar cane of length 63cm is cut into 2 pieces in the ratio 2:7. Find the length of each piece.

6. If the angle measures of a triangle is in the ratio 2:3:4, then find the measure of each angles.

7. Three friends contribute money to a charitable organization in the ratio 3 : 5 : 7. If the largest amount contributed is Birr 35, calculate the total amount of money contributed by the three friends.
Problem Solving

Find the ratio of the areas of the squares PQRS to that of ABCD where AB = 10cm and RQ = 5cm

3.1.2 Proportion

Activity 3.3

Look at the picture to the right; it has eight sections in which counters are labeled.

Place counters in sections V, W and Z so that the ratio of the counters in each column is equal to the ratio of the counters in C and D

Have you ever heard an advertisement on TV like the following?

“8 out of 10 customers prefer Ethiopian coffee.”

It is an expression in the form of ratio, which is equivalent with the fraction \( \frac{8}{10} \). This statement says that \( \frac{8}{10} \) of the customers surveyed prefer Ethiopian coffee to other brands of coffee. Most likely, more than 10 customers were surveyed. Suppose 100 customers were surveyed, then you can find an equivalent fraction with a denominator 100.

\[
\frac{8}{10} = \frac{8 \times 10}{10 \times 10} = \frac{80}{100}
\]

Thus, you can estimate that 80 customers out of 100 prefer the Ethiopian coffee. So, the two fractions \( \frac{8}{10} \) and \( \frac{80}{100} \) are equivalent.
Proportion is a relationship between two quantities (or variables) in which one is a constant multiple of the other. An equation stating that two ratios are equivalent is called a proportion. Two ratios a:b and c:d are equivalent if and only if ad=bc, for c,d≠0.

**Definition:**

A proportion is the equality of two ratios.

In the previous example, the ratio \( \frac{8}{10} \) and \( \frac{80}{100} \) are equivalent. Thus, these ratios are proportional. One can construct other ratios which are equivalent to \( \frac{8}{10} \) by multiplying by a certain factor. Look at the following ratios that are equivalent to \( \frac{8}{10} \).

\[
\begin{align*}
4 & \quad 16 & \quad 24 & \quad 32 & \quad 40 & \quad 88 & \quad 96 \\
5 & \quad 20 & \quad 30 & \quad 40 & \quad 50 & \quad 110 & \quad 120 & \quad \text{etc.}
\end{align*}
\]

**Note**

In the proportion,
\[
\frac{8}{10} = \frac{8 \times 10}{10 \times 10} = \frac{80}{100}, \quad 10 \text{ is a factor}
\]
(constant of proportionality).

**Example 3.9**

Show that the ratios 5:8 and 10:16 form a proportion.

**Solution:**

Rewrite the ratios as fractions and check one is a constant multiple of the other.

a. The fraction form of 5:8 and 10:16 are \( \frac{5}{8} \) and \( \frac{10}{16} \). We have

\[
\frac{5}{8} = \frac{5 \times 2}{8 \times 2} = \frac{10}{16}
\]

Therefore, \( \frac{5}{8} \) and \( \frac{10}{16} \) form a proportion.

Consider the proportion:

\[
\frac{a}{b} = \frac{c}{d}
\]

\[
\frac{a}{b} \times \frac{bd}{bd} = \frac{c}{d} \times \frac{bd}{bd} \quad \text{Multiply each side by } bd.
\]

\[ad = bc \quad \text{Simplify.}\]

The products \( ad \) and \( bc \) are called the cross products of this proportion. The cross products of any proportion are equal. You can compare cross products to identify proportional relationships.
Example 3.10

Show that the ratios 11:7 and 5:3 do not form a proportion.

**Solution:**

The fraction form of 11:7 and 5:3 are \( \frac{11}{7} \) and \( \frac{5}{3} \). But, \( 11 \times 3 = 33 \neq 35 = 7 \times 5 \),

Thus, \( \frac{11}{7} \neq \frac{5}{3} \). Hence, \( \frac{11}{7} \) and \( \frac{5}{3} \) do not form a proportion.

You can also use cross products to find a missing value in a proportion.

Example 3.11

Find the unknown terms in the given proportions

- a. \( 15 : 12 = 35 : x \) (provided \( x \neq 0 \))
- b. \( (3x + 6) : 5 = (x+8) : 3 \)

**Solution:**

From equality of ratios, we have:

a. From \( 15:12 = 35: x \) we have \( \frac{15}{12} = \frac{35}{x} \).

\[
15x = 12 \times 35 = 420 \\
x = 28
\]

Find the cross products

Divide both sides by 15

Therefore, the value of \( x \) is 28.

b. From \( (3x+6):5=(x+8):3 \), we have \( \frac{3x + 6}{5} = \frac{x + 8}{3} \)

\[
3(3x+6) = 5(x+8) \\
9x+18 = 5x+40 \\
4x = 22
\]

Find the cross products

Use distributive property

Divide both sides by 4

Therefore, the value of \( x \) is 5.5.

Example 3.12

In one species, a 6-foot crocodile has a 2-foot skull. If skull length is proportional to body length, what is the length of a crocodile of that same species with a 3.5-foot skull?
Solution:
Let \( b \) represent the length of the crocodile with a 3.5-foot skull.

\[
\frac{\text{body length}}{\text{skull length}} = \frac{6 \text{ ft}}{2 \text{ ft}} = \frac{b \text{ ft}}{3.5 \text{ ft}}
\]

Write a proportion.

\[6(3.5) = 2(b)\]

Find the cross products.

\[21 = 2b\]

Divide each side by 2.

\[10.5 = b\]

So, a crocodile with a 3.5-foot skull is about 10.5 feet long.

Exercise 3.3

1. Determine if the quantities in each pair of ratios are proportional. Explain.
   a. 2 adults for 10 children and 3 adults for 12 children
   b. 12 cm by 8 cm and 18 cm by 12 cm
   c. 8 m in 21 seconds and 12 m in 31.5 seconds
   d. 56 birr for 5 pairs of scissors and 71.2 birr for 8 pairs of scissors

2. Determine whether each pair of ratios forms a proportion or not.
   a. \( \frac{2}{3} \) and \( \frac{3}{5} \)
   b. \( \frac{5}{16} \) and \( \frac{20}{64} \)
   c. \( \frac{3}{4} \) and \( \frac{15}{20} \)
   d. \( \frac{7}{12} \) and \( \frac{3}{5} \)

3. Find the unknown terms in each of the following.
   a. \( \frac{y}{133} = \frac{3}{7} \)
   b. \( \frac{0.2}{3} = \frac{3}{d} \)
   c. \( \frac{3}{n} = \frac{2.7}{18} \)
   d. \( \frac{2}{6} = \frac{5}{h} \)
   e. \( \frac{4}{11} = \frac{a}{132} \)
   f. \( \frac{6}{k} = \frac{24}{28} \)
   g. 10:k=2.5:4
   h. 18:4=y:8

4. Find the fourth proportional term to the following:
   a. 15,12,35
   b. \( a^2, ab, b^2 \)

5. A man saves Birr 48.60 in 9 days. How much will he save in 30 days, if he saves equal amount in all days?

6. Haile Gebre Silassie can run 1,200 meters in 240 seconds. If he runs at the same rate, how many seconds will it take him to run a 3,000-meter race?
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7. Assume that everyone works at the same rate, how long will 20 men take to do a work that 10 men can do in 16 days?

8. There are 2 teachers for 57 students at a certain middle school. If there are 855 students, how many teachers are at the school?

9. If the scale of a certain map is 2:1,000,000. On this map a street is 4cm long. What is the length of the street on the actual land?

10. In a certain school there are 21 teachers and 756 students. By next year, 80 students will leave the school for secondary high school and 152 new students will come. What will be the number of teachers, if the ratio of teachers to students is unchanged?

11. In the proportions, such as \( \frac{3}{6} = \frac{6}{12} \), the same number appear in one of the diagonals (in this case, 6). The repeated number is called the geometric mean of the other two (3 and 12). Find pair of numbers other than 3 and 12 for which 6 is the geometric mean. Repeat the same thing for other geometric mean.

Problem Solving

Project Work: Collect the following data from your school.

- The number of grade 7 and 8 female teachers.
- The number of grade 7 and 8 male teachers.
- The number of administrative staff members in the school.
- The number of all grade 7 and 8 students in the school.

Based on the collected data determine:

a. The ratio of female to male teachers.

b. The ratio of male teachers to the total number of teachers.

c. The ratio of administrative staff to teachers.

d. The ratio of all teachers to all students in the school.

3.1.3 Direct and Inverse (Indirect) Proportionality

Direct Proportionality

Activity 3.4

The following table shows the distance covered by a man riding a bicycle. The time t is given in hours and the distance d is in kilometers.
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<table>
<thead>
<tr>
<th>Time, ( t ) (in hr.)</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Distance, ( d ) (in km.)</td>
<td>10</td>
<td>20</td>
<td>30</td>
<td>40</td>
</tr>
</tbody>
</table>

a. What can you say about the relationship between the time \( t \) and the distance \( d \) covered? What will happen to the distance as the time increases and vice versa?

b. If a man rides 6 hours, how many kilometers would he cover?

c. How much time would he take to cover 80 km?

d. Are distance and time proportional? If so, what is the constant of proportionality?

In the table above, as time, \( t \) increases, distance, \( d \) also increases 10 times. Then, you can write the relationship between \( t \) and \( d \) as \( d = 10t \). Here, 10 is the constant of proportionality.

**Definition:**

We say that \( y \) is directly proportional to \( x \) (written as \( y \propto x \)) if there is a constant \( k, k \neq 0 \) such that \( y = kx \). \( k \) is called the **constant of proportionality**.

**Example 3.13**

If \( y \) is directly proportional to \( x \), and \( y = 6 \) when \( x = 3 \), then find the value of \( y \) when

\[ a. \quad x = 1 \quad b. \quad x = 5 \quad c. \quad x = 24 \]

**Solution:**

Since \( y \propto x \), \( y = kx \) for some constant and \( y = 6 \) when \( x = 3 \). This implies that 6=3k. Thus, \( k = 2 \), which is the constant of proportionality.

\[ a. \quad \text{When } x = 1, \ y = 2x = 2(1) = 2 \]

\[ b. \quad \text{When } x = 5, \ y = 2x = 2(5) = 10 \]

\[ c. \quad \text{When } x = 24, \ y = 2 \cdot 24 = 48 \]

**Example 3.14**

In an area of 5700 m\(^2\), 600 trees can be planted. How many trees could be planted in an area of 0.019 km\(^2\)?
Solution:

Let us represent the problem in a table as follows:

<table>
<thead>
<tr>
<th>Area in m²</th>
<th>5700</th>
<th>0.019 km² = 19,000m²</th>
</tr>
</thead>
<tbody>
<tr>
<td>No. of trees planted</td>
<td>600</td>
<td>Number of trees (T)</td>
</tr>
</tbody>
</table>

Recall that 1km² = 1000m × 1000m = 1,000,000m².

Therefore, 0.019×1,000,000m² = 19,000m².

As area increases, the number of trees to be planted also increases. Therefore, the area and the number of trees is in a direct proportion and the number of trees will be greater than 600.

\[
\frac{600}{T} = \frac{5700}{19000}
\]

Thus, we have \( 5700T = 600 \times 19000 \)

\[
T = \frac{600 \times 19000}{5700} = 2000
\]

Therefore, 2000 trees could be planted in an area of 0.019 m² or 19000 m² of land.

Example 3.15

Suppose \( y \) is directly proportional to \( x \) as shown in the table.

<table>
<thead>
<tr>
<th>x</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>...</th>
<th>q</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>24</td>
<td>32</td>
<td>p</td>
<td>...</td>
<td>72</td>
</tr>
</tbody>
</table>

Find:

a. The constant of proportionality.

b. The value of \( p \).

c. The value of \( q \).

Solution:

Since \( y = 24 = 8 \times 3 \), as \( x = 3 \) and \( y = 32 = 8 \times 4 \) as \( x = 4 \), the constant of proportionality is 8.

Hence \( y = 8x \).

When \( y = p \), \( x = 5 \) means \( p = 8x = 8 \times 5 = 40 \). Similarly, when \( y = 72 \), \( x = q \).

Thus, \( 72 = 8q \), which is \( q = 9 \).

Therefore, the value of \( p = 40 \) and \( q = 9 \).
Inverse(indirect) Proportionality

Activity 3.5

Consider the following two relations:

<table>
<thead>
<tr>
<th>Time (t, in hr.)</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Distance (d, in km.)</td>
<td>5</td>
<td>10</td>
<td>15</td>
<td>20</td>
</tr>
<tr>
<td>( d = 5t )</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Altitude (A, in meter)</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Temperature (T, in °F)</td>
<td>5</td>
<td>2.5</td>
<td>1.67</td>
<td>1.25</td>
</tr>
<tr>
<td>( T = 5 \times \frac{1}{A} )</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

a. What happens to the value of \( d \) as the value of \( t \) increases?

b. What happens to the value of \( T \) as the value of \( A \) increases?

c. What is the constant of proportionality between \( t \) and \( d \)? Between \( A \) and \( T \)?

d. What type of relation is there between temperature and altitude?

Definition:

We say that \( y \) is inversely proportional to \( x \) (written as \( y \propto \frac{1}{x} \)) if there is a constant \( k \), such that \( y = k \times \frac{1}{x} \) or \( xy = k \).

Example 3.16

In the table below, as \( x \) increases, \( y \) decreases. Then, you can write the relationship between \( x \) and \( y \) as \( y = \frac{36}{x} \) or \( xy = 36 \). Here, 36 is the constant of proportionality.

<table>
<thead>
<tr>
<th>( x )</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y )</td>
<td>36</td>
<td>18</td>
<td>12</td>
<td>9</td>
</tr>
<tr>
<td>( xy = 36 = k )</td>
<td>36</td>
<td>36</td>
<td>36</td>
<td>36</td>
</tr>
</tbody>
</table>

Example 3.17

Suppose \( y \) is inversely proportional to \( x \). \( x = 25 \) when \( y = 8 \). Then, what is the value of \( y \) when

a. \( x = 10 \)

b. \( x = 40 \)

c. \( x = 100 \)
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**Solution:**

Since \( y \propto \frac{1}{x} \) and \( x = 25 \) when \( y = 8 \). We have \( y = \frac{1}{x} k \), for some constant \( k \). Then it follows that \( 8 = \frac{k}{25} \)

Which gives, \( k = 8 \times 25 = 200 \)

\[
\begin{align*}
\text{When } x=10, \quad y &= \frac{k}{x} = \frac{200}{10} = 20 \\
\text{When } x=40, \quad y &= \frac{k}{x} = \frac{200}{40} = 5 \\
\text{When } x=100, \quad y &= \frac{k}{x} = \frac{200}{100} = 2
\end{align*}
\]

<table>
<thead>
<tr>
<th>x</th>
<th>10</th>
<th>25</th>
<th>40</th>
<th>100</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>20</td>
<td>8</td>
<td>5</td>
<td>2</td>
</tr>
<tr>
<td>xy</td>
<td>200</td>
<td>200</td>
<td>200</td>
<td>200</td>
</tr>
</tbody>
</table>

**Example 3.18**

If 150 men are working in a factory to produce 6,000 shoes in 15 working days. How long will it take, (assume everyone works at the same rate)

a. 50 men to produce the 6000 shoes?

b. 100 men to produce the 6000 shoes?

**Solution:**

Let us represent the problem in the table as follows:

<table>
<thead>
<tr>
<th>Number of workers (W)</th>
<th>150</th>
<th>100</th>
<th>50</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of days required (D)</td>
<td>15</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

For the same job, as the number of workers (W) decreases, the number of days (D) required will be increased. Thus, \( W \) and \( D \) are inversely related. So, \( WD = K \) for some constant \( k \) and when \( W = 150, \ D = 15 \).

\[
k = WD = 150 \times 15 = 2250
\]

a. \[
D = \frac{k}{W} = \frac{2250}{100} = 22.5 \text{ days}
\]

Therefore, 100 men need 22.5 days to produce 6,000 shoes.
b. \[ \frac{k}{W} = \frac{2250}{50} = 45 \]

Therefore, 50 men need 45 days to produce 6,000 shoes.

Exercise 3.4

1. In each of the following cases, identify which are inversely proportional and which are directly proportional?
   a. The number of people assigned to clean a school compound and the time taken to clean it.
   b. The distance covered at a constant speed and the time taken.
   c. The number of children in a family and the share of their father’s wealth
   d. Perimeter and area of plane figures

2. Suppose \( E \) is directly proportional to \( F \). If \( E \) is 24, then \( F \) is 6.
   a. Find the constant of proportionality
   b. What is the value of \( E \) when \( F \) is 24?

3. Suppose \( p \propto q \). If \( p = 15 \) and the constant of proportionality is 2, then find the value of \( q \).

4. If \( y \) is directly proportional to \( x \), and when \( x = 2 \), then \( y = 18 \). Find \( x \) when:
   a. \( y = 9 \)  
   b. \( y = 45 \)  
   c. \( y = 27 \)

5. Suppose \( A \) is inversely proportional to \( B \). If \( A \) is 4, then \( B \) is 9.
   a. Find the constant of proportionality.
   b. What is \( A \) when \( B \) is 54?

6. \( y \) is inversely proportional to \( x \) as shown in the table below.

<p>| | | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>( x )</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>6</td>
<td>( q )</td>
</tr>
<tr>
<td>( y )</td>
<td>40</td>
<td>30</td>
<td>( p )</td>
<td>20</td>
<td>( \ldots )</td>
</tr>
</tbody>
</table>

Find
   a. the constant of proportionality.
   b. the value of \( p \).
   c. the value of \( q \).
1. For a fixed amount of money, the number of books (N) you can buy is inversely proportional to the price of the book (P). If you can buy 15 books at a price of Birr 20 each.
   a. Make a formula connecting numbers of books (N) bought and the price (P) of books.
   b. Use this formula to find the number of books you can buy costing Birr 6 each.
   c. Find the price of each book, if you can buy 12 of them.

2. A car takes 4 hours to cover some distance at an average speed of 85km/hr. How much time will it take to cover the same distance at an average speed of 60km/hr?

3.2 Revision on Percentages

Do you remember what you have learnt in grade 5 and 6 about decimals, percentage and the conversion of decimals to percentage and vice versa? In this section you will consolidate what you have learned in lower grades about percentage and conversions to, and from decimals.

Activity 3.6

1. In a survey of a market, 28.6% of the customers said that Orange is their favorite fruit. What does 28.6% indicate?

2. In your class, express the ratio of number of female students to the total number of students, first as decimal and then multiply by 100.
   a. What does the result mean?
   b. Do the same for the ratio of number of male students to the total number of students.
   c. Compare the two results.

Percentage is a useful way of making comparisons, apart from being used to calculate many taxes that we pay, such as, income tax, insurance tax, VAT, profit and loss, Turn over Tax, etc. So, percentages are very much part of our lives. We will see about taxes later.
The word percentage and percent are closely related to each other. The word percent means “per one hundred”. For example, 75% of the surface of earth is covered by water that is 75 square units of the earth out of 100 square units are covered by water. Percentage is a measure of a portion in relation to a whole. For example, 75% (75 percent) of 40 students in a class is 30. Here 30 is the percentage.

Example 3.19

Look at the larger square below, it is divided into 100 small squares of equal size. How much portion of the square is shaded?

Solution:

In the first square only 2 small squares are shaded out of 100 squares. So, the shaded portion \( \frac{2}{100} = 0.02 = 2\% \). Similarly, in the second square, the shaded portion accounts \( \frac{20}{100} = 0.20 = 20\% \).

The shaded part is 2 out of 100. That is 2%.

The shaded part is 20 out of 100. That is 20%.

Activity 3.7

1. Explain how to convert
   a. Fraction to decimal
   b. Decimal to percent
   c. Percent to fraction and the vice versa

2. Convert each of the following to other forms.
   (See the figure to the right)
   a. 74%
   b. 0.035
   c. \( \frac{24}{5} \)
Fractions, ratios, percent and decimals are interrelated with each other. Let us look at the conversion of one form to the other;

<table>
<thead>
<tr>
<th>Forms</th>
<th>Description</th>
<th>Examples</th>
</tr>
</thead>
</table>
| Percent to Fraction | Write down the percent divided by 100 and simplify to lowest term | $38\% = \frac{38}{100} = \frac{19}{50}$  
                  |                                                                  | $0.15\% = \frac{0.15}{100} = \frac{15}{10000} = \frac{3}{2000}$ |
| Percent to Decimal | Removing the percent sign and moving the decimal point 2 places to the left | $31\% = 0.31$  
                  |                                                                  | $2.13\% = 2.13$  
                  |                                                                  | $0.5\% = 0.005$ |
| Decimal to Percent | Multiplying the decimal value by 100 and adding the symbol % or by moving the decimal point 2 places to the right and adding symbol % | $0.675 = 0.675 \times 100 = 67.5\%$ |
| Fraction to Percent | Dividing the numerator by the denominator, then multiplying the result by 100 and put the symbol %. | $\frac{3}{10} = 0.3 = 0.3 \times 100\% = 30\%$  
                  |                                                                  | $\frac{2}{5} = 0.4 = 0.4 \times 100\% = 40\%$ |

**Example 3.20**

Look at the following conversion:

<table>
<thead>
<tr>
<th>Ratio</th>
<th>Fraction</th>
<th>Decimal</th>
<th>Percent</th>
</tr>
</thead>
<tbody>
<tr>
<td>1:1</td>
<td>$\frac{1}{1}$</td>
<td>1</td>
<td>100%</td>
</tr>
<tr>
<td>1:3</td>
<td>$\frac{1}{3}$</td>
<td>0.33…</td>
<td>33%</td>
</tr>
<tr>
<td>1:4</td>
<td>$\frac{1}{4}$</td>
<td>0.25</td>
<td>25%</td>
</tr>
<tr>
<td>1:100</td>
<td>$\frac{1}{100}$</td>
<td>0.01</td>
<td>1%</td>
</tr>
<tr>
<td>3:5</td>
<td>$\frac{3}{5}$</td>
<td>0.60</td>
<td>60%</td>
</tr>
</tbody>
</table>
Exercise 3.5

1. Convert each of the following percent to decimals.
   a. 19.8%       b. 628%       c. 77.7%       d. 0.045%

2. Write each of the fractions below as percent.
   a. \( \frac{6}{5} \)       b. \( \frac{5}{9} \)       c. \( \frac{2}{5} \)       d. \( \frac{3}{4} \)

3. Convert each of the decimals to percent.
   a. 0.012       b. 0.006       c. 1.002       d. 24.02

4. Write the following as a fraction in lowest terms.
   a. 2%       c. 112%       e. 0.28
   b. 72%       d. 124.05       f. 0.0024

5. Express the first quantity as a percent of the second.
   a. 100m, 20km       c. 3g, 6g
   b. 20cm, 2m       d. 30 seconds, 1 hour

6. Give at least five fractions which are equal to 150%

How to Calculate Percentage (P)?

Portion of a certain quantity can be expressed as percentages. For example, to know the percentage of female students in your class, you can multiply percent of females by the total number of students in the class.

Activity 3.8

A student obtained the following marks in four tests:

Mathematics 15 out of 20       English 17 out of 25
General Science 33 out of 50       Amharic 7 out of 10

a. What was her mark in percent for each subject?

b. In which subject she did the best?

Problems involving percentage are solved in terms of the basic equation which is given by the formula.

\[ P = \frac{R}{100} \times B, \text{ where } P = \text{Percentage}, R = \text{Rate} \text{ and } B = \text{Base} \]
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Note: “percent (%)” means “per 100”.

Percentage is a measure of portion (part) in relation to a whole.

Example 3.21

Suppose that house furniture is sold at a price of Birr 60,000, and the seller pays 15% of the price for taxes. Then, identify the base, the rate, and the percentage.

Solution:
- the price of the furniture (Birr 60,000) is the base (B),
- the tax (15%) is the rate (R),
- the amount (15% of 60,000), which is calculated as: \( \frac{15}{100} \times 60,000 = 9,000 \) Birr is the percentage (P).

Example 3.22

Calculate the percentage of 60% of Birr 165

Solution:
First identify the base and the rate of each
Given R=60% and B=165.

Then \( P = RB = 60\% \times 165 = \frac{60}{100} \times 165 = \frac{9,900}{100} = 99 \)

Example 3.23

A barber earns 640 Birr in a week. She wants to save 15% of her money. What amount of money will she save?

Solution:
Given that the amount of money she earns (B) = 640, the percent she wants to save (R) = 15%. Required: the amount of money she saves (P)

\[ P = RB = 15\% \times 640 \text{Birr} = \frac{15}{100} \times 640 \text{Birr} = 96 \text{Birr} \]

Therefore, the barber saves 96 Birr in a week.

Note:
There is also another method of finding the percentage of a given percent. We call it percent bar model.
Example 3.24

Abdu has 70 Birr and Ejigu has 30% more money than Abdu. How much money does Ejigu have?

**Solution:**

Consider the bar representing Abdu’s money act as a base, 100%, as Ejigu’s money is based on the relation with the money that Abdu has. In this case, Ejigu has 30% more money than Abdu. Therefore, the amount of money Ejigu has is 130% of the money Abdu has as illustrated below:

![Diagram showing 130% of Abdu's money].

130% is 130 units out of 100, and 100 units are equivalent to the 70 Birr that Abdu has.

\[
100 \text{ units} = 70 \text{ Birr} \\
1 \text{ unit} = \frac{70}{100} \text{ Birr} = 0.7 \text{ Birr} \\
130 \text{ units} = 130 \times 0.7 \text{ Birr} = 91 \text{ Birr}
\]

Therefore, Ejigu has 91 Birr.

Example 3.25

Genet has 50 books and Zemene has 20% fewer books than Genet. How many books does Zemene have?

**Solution:**

We know how many books Genet has, so we can use that amount as a base. Thus, 50 books are 100%. Then, since Zemene has 20% fewer books than Genet, Zemene has 80% of the books that Genet has. Look the percent bar:

![Diagram showing 80% of Genet's books].

100 units = 50 books
1 unit = \frac{50}{100} \text{ books} \\
= 0.5 \text{ books} \\
80 \text{ units} = 80 \times 0.5 \text{ books} = 40 \text{ books} \\
50 \text{ books} - (50 \text{ books} \times 20\%) = 40 \text{ books} \\
Therefore, Zemene has 40 books

**Exercise 3.6**

1. Calculate the percentage of each of the following.
   a. 60\% of Birr 165 
   b. 4.8\% of 3.6 liters 
   c. 90\% of 250 tones 
   d. 15\% of 3240km
2. What percent of
   a. 200 is 65 
   b. 125kg is 15kg 
   c. 2hrs is 45minutes 
   d. 120cm is 12cm
3. If you copy a picture on the photocopying machine at 85\%, what can you say about its size? Enlarging or reducing the picture? Explain.
4. Is a 20\% discount on a 35Birr item the same as a 35\% discount on a 20 Birr item? Explain.
5. 20\% of an article is damaged and thrown away and only 20kg is left. Find its original weight.
6. A retailer agreed to take 5,000 ballpoint pens. However, he found that 12\% are faulty. What was the percentage decrement?
7. One day a student found a wallet containing Birr 3,500 lying on the sidewalk. He turned it into the police. The owner of the money gave him 2\% of the money as a reward. How much money did he get?

**How to calculate Base (B)?**

**Activity 3.9**

At the beginning of the New Year, a Garment Factory announces discount of 15\% on shirts and 20\% on a pair of trousers for the student’s uniform.

a. Ato Dagnachew paid Birr 170 for a shirt. What was the original price of the shirt?
b. W/o Emebet paid Birr 394 for a pair of trousers. What was the original price of a pair of trousers?

Calculating the base means determining the number of which a percent is taken to compute the percentage. We know that percentage is part of the base number (or part of the whole). So, from our previous discussion, we can use the formulas below to calculate base.

\[
\text{Base} = \frac{\text{percentage}}{\text{Rate}} \times 100
\]

\[
B = \frac{P}{R} \times 100 \quad \text{where, } P = \text{Percentage}, \ R = \text{Rate} \text{ and } B = \text{Base}
\]

**Note:** \(\frac{P}{B} = \frac{R}{100}\) is called percent proportion.

**Example 3.26**

Calculate the base in each of the following.

a. 16 minutes is 20% of time T hours. What is the value of T?

b. 90 cm is 180% of y cm. what is the value of y?

**Solution:**

First identify the given and required

a. Given: \(P = 16\) minutes and \(R = 20\%\)

   Required: \(B = T\) hours

   \[
   B = \frac{P}{R} \times 100 = \frac{16\text{ minutes}}{20} \times 100 = 80\text{ minutes}
   \]

   Therefore, the value of T is 80 minutes or 1 hour and 20 minutes.

b. Given: \(P = 90\) cm and \(R = 180\%\).

   Required: \(B = y\) cm

   \[
   B = \frac{P}{R} \times 100 = \frac{90\text{ cm}}{180} \times 100 = 50\text{ cm}
   \]

   Therefore, the value of y is 50 cm.
Example 3.27

A business man saves 15% of what he earns weekly. If he saves Birr 2545 in a week, how much does he earn in a week?

Solution:

Note that 15% is the rate (percent) and Birr 2545 Birr is the percentage, so the required quantity is the base. Then

\[ B = \frac{P \times 100}{R} = \frac{2545 \times 100}{15} = 16,966.66 \text{ Birr} \]

Therefore, the business man earns 169,966.66 Birr in a week.

Exercise 3.7

1. Ayelu saves 20% of what she earns. If she saves Birr 400 a month, how much does she earn a month?

2. If 25% of Teshome’s salary is Birr 1350, what is the amount of his full salary?

3. In a class where the number of girls is 52% of the total number, there are 24 boys. How many students are there in the class?

4. An alloy contains 26% of copper. What quantity of alloy is required to get 260 gram of copper?

5. In a Mathematics club meeting 96% of members were attending the meeting but 5 students were absent. How many students were members of the club?

How to calculate Percent/rate(R)?

Activity 3.10

Consider the number of students in your class.

a. What percent of the students are female?

b. What percent of the students are male?

c. What can you say about sum of percent of female and male?

Rate is the ratio of percentage (amount) to the base. It is written as a percent. Percent/rate is one way of comparison.
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\[
\text{percent (Rate)} = \frac{\text{Percentage}}{\text{Base}} \times 100\%
\]

\[R = \frac{P}{B} \times 100\%, \text{ where, } P = \text{Percentage, } R = \text{Rate and } B = \text{Base}\]

**Note:** Percent/rate is ratio of percentage to base.

**Example 3.28**

Calculate the rate (percent) in each of the following.

a. 400 gm to 2kg
b. Birr 0.75 to Birr 50

**Solution:**

Let us identify the base and percentage

a. Given that \( P = 400 \text{ gm } \) and \( B = 2kg = 2,000\text{gm} \). Required: \( R = ? \)

\[
R = \frac{P}{B} \times 100\% = \frac{400\text{gm}}{2\text{kg}} \times 100\% = \frac{400\text{gm}}{2000\text{gm}} \times 100\% = 20\%
\]

Therefore, the required percent is 20%.

b. Given that \( P = 0.75 \text{ Birr} \) and \( B = 50 \text{ Birr} \). Required \( R = ? \)

\[
R = \frac{P}{B} \times 100\% = \frac{0.75 \text{ Birr}}{50 \text{ Birr}} \times 100\% = 1.5\%
\]

Therefore, the required percent is 1.5%.

**Example 3.29**

A factory has 1200 workers of which 720 are male and the rest are female. What percent of the workers are female?

**Solution:**

There are 1200 workers in a factory. Let male workers and female workers be \( m \) and \( f \) respectively. Then \( m + f = T \)

\[
720 + f = 1200 \quad \text{(Base)}
\]

\[
f = 1200 - 720 = 480 \quad \text{(Percentage)}
\]

\[
R = \frac{P \times 100\%}{B} = \frac{480 \times 100\%}{1200} = 40\%
\]

Therefore, female workers are 40% of the total workers.
Example 3.30

From 165 grade 7 students, 26 of them have got excellent marks on the last test. About what percent of the class got excellent mark?

**Solution:**

**Given:** total number of students \((B) = 165\) and number of students who got excellent marks \((P) = 26\).

Then, percent of students \((R)\)

\[ R = \frac{P \times 100}{B} = \frac{26 \times 100}{165} = 15.76\% \]

Therefore, about 16% of the students got excellent marks.

---

Exercise 3.8

1. Find:
   
   a. 30% of 6  
   
   b. 24% of 120  
   
   c. 125% of 100  
   
   d. 1.8% of 250

2. Seidu earns Birr 6000 per month. He spends 40% of his salary on rent and 15% of his salary on groceries. How much money does Seidu have left for other expenses?

3. Last year Abebe’s salary was Birr 5000. If he gets a 10% increment this year, what is his present salary?

4. The sugar factory produces 4,000 kg in a week. 5% of the product was given as bonus for the workers. Find the amount of sugar that is not given as bonus.

5. Ato Ephrem used to save Birr 300 per month. After his salary increased, he saves Birr 460 per month. Calculate the percent increase in his monthly saving.

6. Bekalu spends 300 Birr for transportation in a month. If his salary is 6000 Birr, what is his percent expenditure?

---

3.3 **Application of Ratio, Proportion and Percentage**

Percent is a convenient and easy way of making comparisons. Business firms use percent to make comparisons of costs and profit in budgeting money of various uses. We use ratio and proportionality to solve a wide variety of percent problems, including problems involving discounts, interest, taxes, and percent increase or decrease.
Application problems that involve percent usually take one of the following forms:

1. Finding a percent of a number
2. Finding what percent of one number is of another
3. Finding a number when a percent of that number is known

**3.3.1 Calculating Profit and Loss as a Percent**

Percentages have a very wide application in every day transactions; one such application is the comparison of business transactions in terms of percent profit or loss.

**Activity 3.11**

1. What is meant by profit or loss? When do businesses get profit? Loss?
2. Consider the following situations
   
   i. Tariku used to fatten sheep for New Year. He bought a sheep for Birr 2000 and after 3 months, he sold the sheep for Birr 3200.

   ii. Abebaw bought a bicycle for Birr 2800. He used it for 2 years and sold for Birr 1450.

      a. Who got a profit? Loss?
      b. How much is their profit or loss?
      c. What is their percent profit or loss?

When goods are bought for a certain amount of money and sold on a different price, there is a gain or loss on the transaction according to the selling price which is more or less than the cost price.

A **profit** is made when the selling price is greater than the cost price. The amount of profit is the difference between the selling price and the cost price. Therefore,

\[
\text{Profit} = \text{Selling Price} (S.P) - \text{Cost Price} (C.P)
\]

To find the profit in percent, you divide the amount of profit by the cost price and multiply by 100%. That is;

\[
\text{Percent Profit} = \frac{\text{Selling Price} - \text{Cost Price}}{\text{Cost Price}} \times 100\%
\]

\[
= \frac{S.P - C.P}{C.P} \times 100\%
\]

\[
= \frac{\text{Profit}}{\text{Cost Price}} \times 100\%
\]
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A loss is made when the cost price is greater than the selling price. The loss is the difference between the cost price and the selling price.

\[
\text{Loss} = \text{Cost Price (C.P)} - \text{Selling Price (S.P)}
\]

Hence, Percent Loss = \[
\frac{\text{Cost Price} - \text{Selling Price}}{\text{Cost Price}} \times 100\%
\]
\[
= \frac{\text{C.P} - \text{S.P}}{\text{C.P}} \times 100\%
\]
\[
= \frac{\text{Loss}}{\text{cost price}} \times 100\%
\]

Example 3.31

A shop keeper buys a jacket for Birr 500 and sells it for Birr 640. Does he make profit or loss? What is the percent profit or loss?

Solution:

Given Cost price (C.P) = Birr 500 and Selling price (S.P) = Birr 640. Since selling price is greater than cost price, he makes a profit.

Then, Percent profit = \[
\frac{\text{Selling price} - \text{Cost price}}{\text{cost price}} \times 100\%
\]
\[
= \frac{\text{Birr 640} - \text{Birr 500}}{\text{Birr 500}} \times 100\%
\]
\[
= \frac{\text{Birr 140}}{\text{Birr 500}} \times 100\% = 28\%
\]

Therefore, the percent profit is 28%.

Example 3.32

A book which costs Birr 150 was sold at 5% discount. What was the selling price?

Solution:

Given: Cost price (C.P) = Birr 150. Percentage loss = 5 and required: Selling price (S.P)

Percent Loss = \[
\frac{\text{C.P} - \text{S.P}}{\text{C.P}} \times 100\%
\]
\[
5\% = \frac{150 - \text{S.P}}{150} \times 100\%
\]
\[
5\% \times 150 = (150 - \text{S.P}) \times 100\%
\]
\[
750 = 15,000 - 100 \times \text{S.P}
\]
\[
100 \times \text{S.P} = 14250
\]
\[
\text{S.P} = \frac{14250}{100} = 142.50 \text{ Birr}
\]

Therefore, the selling price was Birr 142.50
Example 3.33

Zeleke bought 200 eggs for Birr 950 and sold it at Birr 5.25 each. Find his profit percent.

Solution:

Cost price, C.P = Birr 950, Selling price, S.P = 200 × Birr 5.25 = Birr 1050. Since selling price is greater than cost price, he makes profit. Then,

\[
\text{Percent profit} = \frac{S.P - C.P}{C.P} \times 100\% \\
= \frac{1050 - 950}{950} \times 100\% \\
= \frac{100}{950} \times 100\% = \frac{10000}{950}\% = 10.52\%
\]

Therefore, Zeleke’s percent profit is about 10.52%.

Exercise 3.9

1. A merchant made a profit of 17% by selling goods for Birr 175.50. How much was the original price of the goods?

2. A trader bought a TV set for Birr 2000 and sold it at a loss of 5.5%. What was the selling price?

3. If a company’s profit was Birr 1.4 billion in 2010 and Birr 1.8 billion in 2011. What percent of the 2010 profit the company obtained in 2011?

4. A Shopkeeper M sells some goods to N and makes a profit of 15%. N resells to P at a loss of 5%. If P pays Birr 13.11, how much did M pay for the goods?

5. A profit of 24% was made when a book was sold for Birr 34.10; find the selling price that would have given a profit of 28%.

6. A shoe factory produces 800 pairs of shoes a day. Due to material shortage, the factory produced only 240 pairs of shoes a day. Calculate the percent decrease in shoe production.
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3.3.2 Simple Interest

From your previous grades, do you remember what is meant by simple interest?

Activity 3.12

Ayalew wants to start his own business, but he has no enough money to start. His friend, Ibrahim advised him to ask credit from a small-scale finance company. He borrowed Birr 10,000 for 5 years with an agreement that for each 100 Birr there is additional 9 Birr per year.

a. What will be the additional money that he will pay for a year?

b. After the end of 5 years, what will be the total amount of money that he will pay?

When money is lent, particularly for business, the borrower is expected to pay for the use of the money. The amount of money borrowed is called the **principal** and the charge made for the use of the money is called **interest**.

Interest on money borrowed is paid at definite time intervals (monthly, quarterly, half-yearly or yearly). It is usually calculated as a percentage of the principal for the period stated until the loan is repaid.

The interest paid on the original principal only during the whole interest periods is called **simple interest**. Simple interest can be expressed in terms of the basic interest formula as follows:

**Interest = Principal × Rate × Time**

That is, \( I = PRT \)

where, \( I = \) amount of interest,

\( P = \) principal

\( R = \) interest rate per period (expressed as percentage)

\( T = \) time (in years)

**Note:** After a certain time, total amount \( (A) \) is given by \( A = P + I \)

Example 3.34

W/ro Etaferahu is a hard working farmer. She started cultivating crops and fruits by applying new technologies and irrigation. For her farm, she borrowed Birr 300,000 for 6 years at a rate of 11% simple interest per year from the bank.

a. How much interest will she pay for the bank yearly?

b. What will be the amount she will pay at the end of 6 years?
Solution:

Given: $P = 300,000$ Birr, $R=11\%$ and $T = 6$ years

a. One year interest $I$, for $T = 1$ year

$$I = PRT = 300,000 \text{ Birr} \times \frac{11}{100} \times 1 = 33,000 \text{ Birr}$$

Therefore, she will pay $33,000$ Birr yearly as simple interest

b. Total amount after $T = 6$ years.

Total simple interest:

$$I = 33,000 \text{ Birr} \times 6 = 198,000 \text{ Birr}$$

The amount in 6 years:

$$A = P + I = 300,000 \text{ Birr} + 198,000 = 498,000 \text{ Birr}$$

Therefore, she will pay $498,000$ Birr after 6 yearly.

Example 3.35

How long will it take to double Birr 300 if it is invested at the rate of 5% simple interest per year?

Solution:

We know that $I = PRT$, $A = P + I$. Then

$$A = P + PRT = P(I + RT)$$

Then, $600 \text{ Birr} = 300 \text{ Birr} (1 + 0.05T)$

$$2 = 1 + 0.05T$$

$$T = \frac{1}{0.05} \text{ years} = 20 \text{ years}$$

Therefore, it takes 20 years to double Birr 300.

Example 3.36

The simple interest for 9 months at 8% is Birr 37.50. Find the principal.

Solution:

Given: $T = 9$ months = 0.75 year, $R = 8\%$ and $I = \text{Birr 37.50}$

Required: $P =$ ?
I = PRT

\[ P = \frac{I}{RT} = \frac{37.50 \text{ Birr}}{(0.08)(0.75)} = \frac{37.50}{0.06} \text{ Birr} = 625 \text{ Birr} \]

Therefore, the principal is 625 Birr.

**Exercise 3.10**

1. Find the simple interest on
   a. Birr 300 for 3 years at 5%
   b. Birr 525 for 4 years at \( \frac{1}{2} \) %.
2. If the simple interest on a certain sum of money invested at 3.5% per year for 44 years is Birr 420, find the principal.
3. If Birr 12,000 is invested at 10% simple interest per year, then what is the amount after 5.5 years?
4. Find the time in which Birr 168.40 will earn a simple interest of Birr 29.47 at 5% per year.
5. Find the principal which earns a simple interest of Birr 115.38 in 8 years at 4% per year.
6. Find the principal which amounts to Birr 142.83 in 5 years at 3% per year.
7. Find the rate of interest at which Birr 380,000 earns a simple interest of Birr 12,800 in 7 years and 6 months.
8. Over what period of time will Birr 500 amount to Birr 900 at the rate of 8% simple interest?
9. An investor borrows Birr 8 million for 4 years at a simple interest rate of 12%. What is the total amount that must be repaid after 4 years?
10. A worker saves his money in a bank with a simple interest rate of 8% and got Birr 600 interest after 3 years. What was his initial capital?

### 3.3.3 Compound Interest

In business transactions, interest is sometimes calculated daily (365 times a year). In the case of savings, the earned interest is added daily to the principal, and each day the interest is earned on a different amount; that is, it is earned on the sum of the previous interest as well as the principal. Interest earned in this way is compound interest.
Therefore, **compound interest** is an interest that is paid both on the original amount of money saved and on the interest that has been added to it. Compounding is usually done annually (once a year), semiannually (twice a year), quarterly (4 times a year), or monthly (12 times a year).

Though common in the past for banks to compound interest quarterly, today banks may compound interest monthly, daily, or even continuously. However, even when the interest is compounded, it is given as an annual rate. For example, if the annual rate is 6% compounded monthly, the interest per month is \( \frac{6}{12} \) % or 0.5%. If it is compounded daily, the interest per day is \( \frac{6}{365} \) %. In general, the interest rate per period is the annual interest rate divided by the number of periods in a year.

For example, if you invest 100 Birr at 8% annual interest compounded quarterly, then how much will you have in your account after 1 year? The quarterly interest rate is \( \frac{1}{4} \) (8%) or 2%. It seems that you would have to calculate the interest 4 times. If at the beginning of any of the four periods there is x Birr in the account. At the end of that period there will be:

\[
x + 2\% \text{ of } x = x + 0.02x = x(1 + 0.02) = x(1.02) \text{ Birr}
\]

Hence, to find your amount at the end of any period, you need only multiply the amount at the beginning of the period by 1.02.

<table>
<thead>
<tr>
<th>Period</th>
<th>Initial Amount</th>
<th>Final amount</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>100</td>
<td>100×1.02</td>
</tr>
<tr>
<td>2</td>
<td>100×1.02</td>
<td>(100×1.02)×1.02=100×(1.02)^2</td>
</tr>
<tr>
<td>3</td>
<td>100×(1.02)^2</td>
<td>((100×(1.02)^2)×1.02)=100×(1.02)^3</td>
</tr>
<tr>
<td>4</td>
<td>100×(1.02)^3</td>
<td>((100×(1.02)^3)×1.02)=100×(1.02)^4</td>
</tr>
</tbody>
</table>

From the table above, you can obtain the amount at the end of the fourth period is given by

\[
\text{Amount} = 100 \times (1.02)^4 = 100 \times (1+0.02)^4
\]

Principal Interest rate

The calculator displays 108.243216. Thus, the amount at the end of 1 year is approximately 108.24 Birr.

The amount at the end of the nth period is computed as
\[ A = P(1 + \frac{r}{n})^n \]

where, \( A \) = amount at the end of \( n \)th year
\( P \) = principal
\( r \) = interest rate
\( n \) = number of times interest applied per time period
\( t \) = number of time periods elapsed

**Example 3.37**

Suppose you deposit Birr 1000 in a savings account that pays 6% annual interest compounded quarterly. What is the balance at the end of 1st year?

**Solution:**

An annual interest rate of 6% earns \( \frac{1}{4} \) of 6% or an interest rate of \( \frac{0.06}{4} \) in the first quarter as there are 4 periods in a year.

\[ A = \text{Birr} \ 1000 \times \left(1 + \frac{0.06}{4}\right)^4 = \text{Birr} \ 1061.36 \quad \text{... (Use a calculator)} \]

The balance at the end of one year is approximately Birr 1061.36.

**Example 3.38**

A couple deposits Birr 3000 into an account that pays 10% annual interest compounded annually for their child’s college education. Find the total amount after 8 years.

**Solution:**

The principal is Birr 3000, the rate is 10% and the number of compounding period is \( 8 \times 1 = 8 \) years.

Thus we have, \( A = P(1 + \frac{r}{n})^n \)

\[ = 3000 \left(1 + \frac{0.1}{1}\right)^{1 \times 8} \]
\[ = 3000 \times 2.14358 \]
\[ = 6430.76 \]

Therefore, the amount in the account after 8 years is approximately Birr 6430.76
UNIT 3 : RATIO, PROPORTION AND PERCENTAGE

Exercise 3.11

1. Suppose you deposit Birr 100 with 8% interest compounded quarterly. How much amount will you have at the end of 5 years?

2. Ato Samuel borrowed Birr 200,000 for his investment at a rate of 7% compounded annually for 5 years.
   a. What is the compound interest that he should pay at the end of the 5th year?
   b. What is the total amount that he pays at the end of the 5th year?

3. W/ro Aminat deposited Birr 3,000 in a saving account that pays 6% interest rate per year, compounded quarterly. What is the amount of interest obtained at the end of 7 years? (Assuming no deposit or withdrawal is made in these seven years).

4. Find the amount and the compound interest on Birr 10,000 at a rate of 6% for 3 years compounded annually.

Activity 3.13

How can we use Microsoft excel to compute compound interest?

Simple interest, which you studied in section 2.3.2, is calculated only on the initial principal of a savings account or a loan. Compound interest is calculated on the initial principal and on interest earned in the past. You can use a spreadsheet (excel) to investigate the growth of compound interest.

Example 3.39

Find the value of a 3,000 Birr savings account after 5 years if the account pays 8% interest compounded semiannually.

Solution:

8% interest compounded semiannually means that the interest is paid twice a year. The interest rate is 8% ÷ 2 or 4% for each 6 months.

The value of the savings account after five years is Birr 4,440.73
The interest rate is entered as a decimal.

Excel evaluates the formula $A4 \times B1$.

The interest is added to the principal every 6 months. Excel evaluates the formula $A4 + B4$.

**Exercise 3.12**

1. Use a spreadsheet to find the value of a savings account if Birr 3,000 is invested for four years at 8% interest compounded quarterly.

2. Suppose you leave Birr 1,000 in each of three bank accounts paying 6% interest per year. One account pays simple interest, the other pays interest compounded semiannually, and the third pays interest compounded quarterly. Use a spreadsheet to find the amount of money in each account after three years.

3. How does the amount of interest change if the compounding occurs more frequently? Explain your reasoning.

### 3.3.4 Taxes

**Activity 3.14**

Ask your parents about taxes and answer the following questions.

- a. What is tax?
- b. Who collects tax?
- c. For what purpose is tax collected?
- d. From whom is tax collected?
- e. List the different types of taxation and explain their differences.
**UNIT 3 : RATIO, PROPORTION AND PERCENTAGE**

**Tax** is money that one has to pay to the government so that it can be used for public services. As governments have played a growing role in all economies, they have used increasing amounts of resources for their activities. Governments provide: education for the citizen; pay a major proportion of medical bills, national defense, police and fire protection; support a substantial amount of housing, recreation facilities and parklands; ensure adequate water supplies, transportation and other public facilities.

In order to do all the above-mentioned activities, where does the government get money? It is from the taxes collected from the citizens and other income generating activities.

In Ethiopia, there are different types of taxes. Some of these are income tax, Value Added Tax (VAT), Turn Over Tax (TOT).

**Income Tax** is the money deducted from any income of a person like employment, rental of building, income from business etc.

**VAT** (Value Added Tax) is a tax imposed by the government on sales of some selected goods and services. VAT is a charge imposed on business at all levels of production and distribution of goods and services. To apply VAT you add 15% extra onto the original cost.

To enhance fairness in commercial dealings and to make a complete coverage of the tax system, a **Turn over Tax (TOT)** is imposed on those who are not required to register for VAT, but supply goods and services in the country.

The TOT rate is

- On goods sold locally: 2%
- On services rendered locally: contractors, grain mills, tractors and combine harvesters: 2% and others 10%.

---

**Example 3.40**

The price of a machine is Birr 3000 plus 15% VAT. How much is the VAT?

**Solution:**

The VAT is 15%

$$VAT = \left( \text{Birr 3000} \times \frac{15}{100} \right) = \text{Birr 450}$$

Therefore, the VAT is Birr 450.

Thus, a person who buys this machine will pay Birr \((3000 + 450) = \text{Birr 3450}\) in total.
UNIT 3: RATIO, PROPORTION AND PERCENTAGE

Example 3.41

A shoe dealer purchased net Birr 8,000 worth of shoes from a shoe company.

a. Find the amount to be paid to the company including VAT.

b. If the dealer sold the shoes for Birr 12,000 including VAT, find the amount of VAT liable to the dealer.

Solution:

Given: Net price = Birr 8,000, and VAT = 15%

a. Price including VAT = \( \text{Birr} \frac{8,000 + \frac{15}{100} \times 8,000}{8,000} \)
   \[ = \text{Birr} 8,000 + \text{Birr} 1200 \]
   \[ = \text{Birr} 9,200 \]

Therefore, the company will pay including VAT is 9,200 Birr

b. Price including, VAT = 15%

Required Net price (NP) = ?

\[ NP + \frac{15}{100} NP = \text{Birr} 12,000 \]
\[ NP + 0.15NP = \text{Birr} 12,000 \]
\[ NP = \frac{12,000}{1.15} \text{Birr} = 10,434.78 \text{Birr} \]

Thus, the net price is Birr 10,434.78.

Exercise 3.13

1. My mother has bought a washing machine whose price including VAT (value added tax) is Birr 15,750. Its VAT is charged at 15%, how much was the price before VAT?

2. A sales tax of 6% of the cost of a car was added to the purchase price of Birr 600,000. What is the total cost of the car including sales tax?

3. A secretary gets a salary of Birr 5,400 per month. Of this amount 20% is deducted for income tax. Find her net salary.

4. The price of television is Birr 12,000 plus 15% VAT.
   a. How much is the VAT?
   b. How much is the total price including VAT?
Problem Solving

If 15%, VAT is deducted from each of the following item, then what will be the amount of VAT for each item whose selling price is indicated?

a. 125 Birr including VAT
b. 450 Birr excluding VAT
c. 25 Birr excluding VAT

Unit summary

- Two or more quantities can be compared using ratios, proportions and percentages.
- Two ratios form a proportion if the product of the means and the extremes are equal.
- Two quantities are proportional if one is a constant multiple of the other.
- Two quantities are directly proportional if one increases (respectively decreases) the other also increases (respectively decreases).
- Two quantities are inversely proportional if one increases (respectively decreases) the other decreases (respectively increases).
- The relationship among base, rate and percentage is given by
  
  \[
  \text{Percentage} = \frac{\text{Rate} \times \text{Base}}{100}, \quad R = \frac{\text{Percentage} \times 100}{\text{Base}}, \quad \text{Base} = \frac{\text{Percentage} \times 100}{\text{Rate}}
  \]
- The relationship among cost price, profit and loss is given by
  
  \[
  \text{Cost Price} = \frac{\text{Profit} \times 100}{\text{Percentage Profit}} = \frac{\text{Loss} \times 100}{\text{Percentage Loss}}
  \]
- Simple interest is the interest paid only for the original amount of money that is invested or deposited, but not for any interest that it has earned.
- Compound interest is interest paid for both the original amount of money saved and the interest that has been added to it.
- Tax is collected to the government for public services. Its rate depends on the type of services.

Review Exercises

1. If \( \frac{x}{y} = \frac{3}{5} \), then find the value of the expression \( \frac{6y + 5x}{3y - 2x} \).

2. Find the decimal and fraction form of 672.937%.
3. Three numbers d, m and n are in the ratio 3:6:4. Find the value of
   a. \(d-2m:3m-n\)
   b. \(\frac{4d-m}{m+2n}\)

4. Identify the largest of the following
   a. 95% of 200
   b. 105% of 180
   c. 50% of 400
   d. 5% of 4050

5. If \((x + 4), (x + 12), (x - 1)\) and \((x + 5)\) are in proportion, then find the value of \(x\).

6. If \(h:k = 2:5, x:y = 3:4\) and \(2h+x:k+2y = 1:2\), find the ratio \(h-x:k-y\).

7. The ratio of the number of pigs to the number of horses in a farm is 2:3. If there are 24 horses, what is the number of pigs?

8. In a given class room, the number of girls is ten greater than the number of boys. If the ratio of the number of girls to boys is 7:5, then find
   a. the number of girls
   b. the number of boys
   c. the total number of students

9. In a school, 58% of the total number of students are boys. If the number of girls is 840, how many students are there in the school?

10. Birr 300 is invested at 6% simple interest per year. How long will it take to earn Birr 180 interest?

11. If Birr 400 is invested for 4 months with simple interest Birr 12. What is the annual (yearly) rate of interest?

12. In a school the number of girls exceeds the number of boys by 15%. Find the ratio of the number of boys to girls.

13. A shop keeper bought a pair of shoes in Birr 1,500 and sold it in birr 1,800. What is the percentage profit?

14. A merchant gains 15% profit by selling an article for Birr 150. By how much does he sell it to double his profit?

15. A 6% tax on a pair of shoes amounts Birr 40.20. What is the cost of the pair of shoes?

16. Find the compound interest on Birr 5000 at 5% for 4 years compounded annually. What is the total amount at the end of 4 years?

17. Nardos deposited Birr 30,000 in a saving account that pays 7% interest rate per year, compounded quarterly. What is the amount of interest obtained at the end of 6 years? (No deposit or withdrawal is made in these years)

18. A sales tax of 6% of the cost of a car was added to the purchase price of Birr 600,000. What is the total cost of the car including sales tax?
UNIT - 4
LINEAR EQUATIONS

Learning Outcomes:

After completing this unit, you will be able to:

- Identify variables and terms in algebraic expressions
- Simplify algebraic expressions
- Solve linear equations.
- Apply the rules of transformation of linear equations for solving problems
- Draw graph of linear equations.
- Apply the concept of linear equations in solving real life problems

Key Terms

- Terms
- Algebraic expressions
- Linear equation
- Equivalent Equation
- Abscissa
- Slope
- Like and unlike terms
- Numerical coefficient
- Transformation rules
- Properties of equality
- Cartesian coordinate system
- Quadrants
- Ordinate
UNIT 4: LINEAR EQUATIONS

Introduction

You remember that in unit one, you have learnt about properties of basic mathematical operations on integers, like the commutative, associative and distributive properties. These rules are applicable in transforming linear equations to an equivalent equation which is simpler in finding its solution. In this unit, you will learn how to solve linear equations involving more than one step.

4.1 Algebraic Expressions and Terms

Activity 4.1

A box contains some pencils.
There are also three pencils outside of the box.
The total number of pencils is the sum of three and some number.
The three pencils represent the known value,
and the pencils in the box represent the unknown value.
Write a mathematical expression that represents the total number of pencils?

In mathematics, a letter or symbol that is used to represent a number or some unknown quantity is called a variable. The expression $3 + n$ represents the sum of three and some unknown number.

Algebraic expressions are combinations of variables, numbers, and at least one operation.

At least one operation

Any letter can be used as a variable

$3 + n$

Combination of numbers and variables

It is common to use the letter $x$, or the first letter of the word describing the value you are representing, as a variable. The variables in an expression can be replaced with any number. Once the variables have been replaced, you can find the value of, the algebraic expression.
Example 4.1

Evaluate the following Expressions

a. 5\(t\) + 4 if \(t = 3\) 
   \[5t + 4 = 5 \times 3 + 4\]
   = 15 + 4
   = 19

b. \(x - y\) if \(x = 64\) and \(y = 27\).
   \[x - y = 64 - 27\]
   = 37

Solution:

a. 5\(t\) + 4 = 5\times 3 + 4  
   Replace \(t\) with 3.
   = 15 + 4
   Multiply 5 and 3.
   = 19
   Add 15 and 4.

a. \(x - y\) = 64 - 27  
   Replace \(x\) with 64 and \(y\) with 27.
   = 37
   Subtract 27 from 64.

Example 4.2

An expression for finding the area of a triangle whose height is 3 units longer than its base
is \(\frac{1}{2}b(b+3)\), where \(b\) is the measure of the base. Find the area of a triangle with a base 8
units long.

Solution:

You need to find the value of the expression given \(b = 8\).

\[
\frac{1}{2}b(b+3) = \frac{1}{2}8(8+3)
\]
Replace \(b\) with 8.

= 4\times11 = 44

Therefore, the area of the triangle is 44 square units.

When plus or minus signs separate an algebraic expression into parts, each part is called a term. Terms can be a constant, or a variable, or a product of a constant and a variable. The numerical factor of a term that contains a variable is called the coefficient of the variable.
Like terms contain the same variables to the same powers, but can differ in their coefficients. For example, 5x^2 and -6x^2 are like terms. So are x, y and 15x, y. But -11xz^2 and 21x^2z are not like terms. A term without a variable is called a constant. Constant terms are also like terms.

**Example 4.3**

Identify the terms, like terms, coefficients, and constants in the expression $6n - 7n - 4 + n$.

\[ 6n - 7n - 4 + n = 6n + (-7n) + (-4) + n \quad \text{Definition of subtraction} \]
\[ = 6n + (-7n) + (-4) + 1n \quad \text{Identity Property; } n = 1n \]

- Terms: 6n, -7n, -4, n
- Like terms: 6n, -7n, n
- Coefficients: 6, -7, 1
- Constant: -4.

**Example 4.4**

Classify each of the following as like and unlike terms and determine the coefficients.

\[ \frac{1}{2} \ x^2y^2, \ 1045y^2, \ -7pq^2, \ 12p^2q, \ -52y^2, \ 42x^2y^2 \]

**Solution:**

\[ \frac{1}{2} x^2y^2 \text{ and } 42x^2y^2 \text{ are like terms. } \frac{1}{2} \text{ and } 42 \text{ are their numerical coefficients respectively.} \]

1045y^2 and -52y^2 are like terms with numerical coefficients 1045 and -52 respectively.

-7pq^2 and 12p^2q are unlike terms. -7 and 12 are the respective numerical coefficients.

**Exercise 4.1**

1. Identify each of the following pairs of terms as like terms or unlike terms.
   a. 3x and 25x
   b. -4x^3 and -3x^4

2. Identify the terms, like terms, coefficients, and constants in each expression.
   a. 3y - 9 - 13y + 5
   b. 7x + 4 - 11 - 2x
   c. 8n - 3n - 3 + n
   d. 4y + 4 - 6y - 5y
   e. -3d + 8 - d - 2
   f. n + 3n - 9n - 1
3. Evaluate each expression if \( p = -8 \) and \( q = 9 \).

   a. \( p + 9 \)  
   c. \( p \div 4 \)  
   e. \( 6 \times p \)  
   g. \( 4p - 2 \)  
   b. \( 9 - p \)  
   d. \( 18 \div q \)  
   f. \( q + p \)  
   h. \( 2q + 3 \)  

4. Distance traveled can be found using the expression \( rt \), where \( r \) represents the speed (rate) and \( t \) represents time. How far did a hot air balloon travel at an average rate of 15 miles per hour for 6 hours?

### 4.1.1 Simplification of Algebraic Expressions

**Activity 4.2**

You can use algebra tiles to rewrite the algebraic expression \( 2(x + 3) \).

Represent \( x + 3 \) using algebra tiles.  
Double this amount of tiles to represent \( 2(x + 3) \).  
Rearrange the tiles by grouping together the ones with the same shape.

1. Choose two positive and one negative value for \( x \). Then evaluate \( 2(x + 3) \) and \( 2x + 6 \) for each of these values. What do you notice?

2. Use algebra tiles to rewrite the expression \( 3(x - 2) \).  
   (Use one blue \( x \)-tile and two red \((-1)\)-tiles to represent \( x - 2 \).)

In the previous chapter, you learned that expressions like \( 2(6 + 3) \) can be rewritten using the distributive property as: \( 2(6 + 3) = 2(6) + 2(3) = 12 + 6 \) or 18.

The distributive property can also be used to simplify an algebraic expression like \( 2(x + 3) \).

\[
2(x + 3) = 2(x) + 2(3) = 2x + 6.
\]

The expressions \( 2(x + 3) \) and \( 2x + 6 \) are equivalent expressions, because no matter what \( x \) is, these expressions have the same value.
Example 4.5

Use the distributive property to rewrite each expression.

a. \(4(m - 3)\)

\[\begin{align*}
4(m - 3) &= 4[m + (-3)] & \text{Rewrite } m - 3 \text{ as } m + (-3). \\
&= 4m + 4(-3) & \text{Distributive Property} \\
&= 4m + (-12) \\
&= 4m - 12 & \text{Definition of subtraction}
\end{align*}\]

b. \(-5(n - 7)\)

\[\begin{align*}
-5(n - 7) &= -5[n + (-7)] & \text{Rewrite } n - 7 \text{ as } n + (-7). \\
&= -5(n) + (-5)(-7) & \text{Distributive Property} \\
&= -5n + 35
\end{align*}\]

An algebraic expression is in **simplest form** if it has no like terms and no parentheses. You can use the distributive property to remove the parenthesis and combine like terms. This is called **simplification of the expression**.

Example 4.6

Simplify the expressions

a. \(4x + x\)

\[\begin{align*}
4x + x &= 4x + 1x & \text{Identity Property; } x = 1x \\
&= (4 + 1)x & \text{Distributive Property.} \\
&= 5x & \text{Simplify.}
\end{align*}\]

b. \(11y - 6 - 11y + 9\)

\[\begin{align*}
11y - 6 - 11y + 9 &= 11y + (-6) + (-11y) + 9 & \text{Definition of subtraction} \\
&= 11y + (-11y) + (-6) + 9 & \text{Commutative Property} \\
&= [11 + (-11)]y + (-6) + 9 & \text{Distributive Property} \\
&= 0y + 3 & \text{Simplify.} \\
&= 0 + 3 \text{ or } 3 & 0y = 0.
\end{align*}\]
Example 4.7

Simplify: \(7(x-3)-2(x+4)\)

Solution:

\[
7(x - 3) - 2(x + 4) = 7x - 21 - 2x - 8
\]
\[
= 7x - 2x - 21 - 8
\]
\[
= 5x - 29
\]

Example 4.8

At school annual celebration day, you buy some soft drinks that cost 14 Birr each and the same number of biscuits for 9.50 Birr each. Write an expression in simplest form that represents the total amount spent.

Solution:

You spent 14 Birr each for some number of soft drinks, and 9.50 Birr each for the same number of biscuits.

Let \(x\) represent the number of soft drinks or biscuits.

\[
14x + 9.50x.
\]

\[
14x + 9.50x = (14 + 9.50)x \quad \text{Distributive Property}
\]

\[
= 23.50x \quad \text{Simplify.}
\]

The expression 23.50x Birr represents the total amount you spent.

Exercise 4.2

1. Use the distributive property to rewrite each of the following expressions.
   a. \(4(x + 3)\)
   b. \((a + 11)5\)
   c. \(-7(y + 1)\)
   d. \(2(t - 8)\)
   e. \(-9(w - 9)\)
   f. \((m - 2)(-7)\)
   g. \(-6(n - 4)\)
   h. \(-3(p + 12)\)

2. Simplify each of the following algebraic expressions.
   a. \(8x - x\)
   b. \(15 - 3t + 3t\)
   c. \(5q - 3 + 10 - 6q\)
   d. \(-7y + y + 1\)
   e. \(7n + 5 - 2n + n\)
   f. \(9p - 12 + 9p + 12\)

3. Simplify each of the algebraic expressions.
   a. \(3(x+2)+7(x+8)\)
   b. \(2(p+5)-(p-4)\)
UNIT 4: LINEAR EQUATIONS

c. \(3(2-x)+7(x-3)\)  
e. \((q-4)-(3-q)\)

d. \(-35p+75-4(p+1)\)  
f. \(4(m-6)-2(m+1)\)

4. You buy 2 soft drinks that each cost \(x\) Birr and a large loaf of bread for 12.50 Birr. Write an expression in simplest form that represents the total amount of money you spent.

5. You have saved some money. Your friend has saved 250 Birr less than you. Write an expression in simplest form that represents the total amount of money you and your friend have saved.

4.2 Solving Linear Equations

4.2.1 Writing Expressions and Equations

Activity 4.3

Earth has only one moon, but other planets have many moons.

For example, Uranus has 21 moons, and Saturn has 10 more moons than Uranus.

1. What operation would you use to find how many moons Saturn has? Explain.

2. Jupiter has about three times as many moons as Uranus. What operation would you use to find how many moons Jupiter has?

Words and phrases in problems often suggest addition, subtraction, multiplication, and division. Here are some examples

<table>
<thead>
<tr>
<th>Addition and Subtraction</th>
<th>Multiplication and Division</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sum</td>
<td>difference</td>
</tr>
<tr>
<td>more than</td>
<td>less than</td>
</tr>
<tr>
<td>increased by</td>
<td>Less by</td>
</tr>
<tr>
<td>in all</td>
<td>decreased by</td>
</tr>
<tr>
<td></td>
<td>twice</td>
</tr>
</tbody>
</table>

An equation is formed when two algebraic expressions are joined by the equality sign (=). When you write a verbal sentence as an equation, you can use the equals sign (=) for the words equals or is.
Example 4.9

Write each sentence as an algebraic equation.

a. Six less than a number is 20.
   \[ n - 6 = 20. \]

b. Three times Kebede’s age equals 12.
   \[ 3a = 12. \]

Solution:

a. Let \( n \) represent the number.
   \[ n - 6 = 20. \]

b. Let \( a \) represent Kebede’s age.
   \[ 3a = 12. \]

Example 4.10

Write an equation that models the situation.

A giraffe is 3.5 meters taller than a camel. If a giraffe is 5.5 meters tall, how tall is a camel?

Solution:

A giraffe is 3.5 meters taller than a camel.
Let \( h \) represent the height of the camel.
Thus, \( 5.5 = 3.5 + h \).
The equation is \( 5.5 = 3.5 + h \).
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g. The sum of a number and four is equal to -8.
h. The product of a number and five is -20.

3. Express each of the given expressions in at least three different verbal phrases.
   a. x - 10  
   b. 10y  
   c. 4 + q  
   d. \( \frac{p}{3} \)

Activity 4.4

Mahlet had some chewing gums, and then she bought three more gums. Now she has six gums.

1. What does \( x \) represent in the figure?
2. What addition equation is shown in the figure?
3. Explain how to solve the equation.
4. How many chewing gums did Mahlet have at the beginning?

You can solve the equation \( x + 3 = 6 \) by removing, or subtracting, the same number of positive tiles from each side of the box. Similarly, you can also solve the equation without using models or a drawing. You can subtract 2 from each side of the equation. The variable is now on one side of the equation.

\[
\begin{align*}
\text{Use Models} & \\
\text{Use Symbols} & \\
\hline
x + 3 &= 6 \\
-3 &= -3 \\
x &= 3
\end{align*}
\]

Subtracting 3 from each side of an equation illustrates the Subtraction Property of Equality.

Note:

If you subtract the same number from each side of an equation, the two sides remain equal.

That is: if \( a = b \), then \( a - c = b - c \).
Example 4.11

Solve: $x + 7 = 3$. Check your solution.

Solution:

\[
x + 7 - 7 = 3 - 7 \\
x = -4
\]

Check: $x + 7 = 3$

\[
-4 + 7 = ? \\
\checkmark 
\]

The solution is $-4$.

Example 4.12

The highest recorded temperature in Dessie, is $77^\circ F$. This is $31^\circ F$ greater than the lowest recorded temperature. Write and solve an equation to find the lowest recorded temperature.

Solution:

The highest recorded temperature is $31^\circ F$ greater than the lowest recorded temperature.

Let $x$ represents the lowest recorded temperature.

Then, the equation is: $77 = 31 + x$.

Now solve the equation.

\[
77 = 31 + x \\
-31 = -31 \\
46 = x
\]

The lowest recorded temperature in Dessie is $46^\circ F$.

Similarly, you can use inverse operations and the Addition Property of Equality to solve subtraction equations.

Note:

If you add the same number to each side of an equation, the two sides remain equal.

That is: if $a = b$, then $a + c = b + c$.

Example 4.13

Solve: $x - 13 = 25$. 
Solution:

\[ x - 13 + 13 = 25 + 13 \]. Add 13 on both sides.

\[ x = 38. \]

The solution is 38.

Equations like \( 3x = 6 \) are called multiplication equations because the expression \( 3x \) means \( 3 \text{ times the value of } x \). So, you can use the Division Property of Equality to solve multiplication equations.

**Note:**

If you divide each side of an equation by the same nonzero number, the two sides remain equal.

That is: if \( a = b \) and \( c \neq 0 \), then \( \frac{a}{c} = \frac{b}{c} \).

**Example 4.14**

Solve: \(-7x = 28\).

**Solution:**

\[-7x = 28\]

Write the equation.

\[-\frac{7x}{-7} = \frac{28}{-7}\]

Divide each side by \(-7\).

\[ y = -4 \]

\[ 28 \div (-7) = -4 \]

Therefore, the solution is \(-4\).

**Exercise 4.4**

1. Solve each equation and check your solution.

   a. \( x + 5 = 16 \)  
   d. \( y - 3 = 4 \)  
   g. \( 8 = n + 3 \)  
   j. \( y - 4 = 7 \)

   b. \( y + 7 = 1 \)  
   e. \( r - 5 = -3 \)  
   h. \( t + 20 = 11 \)  
   k. \( -1 = p - 9 \)

   c. \( -3 = p + 6 \)  
   f. \( q - 9 = -13 \)  
   i. \( -3 = b + 7 \)  
   l. \( -2 = q - 12 \)

2. Solve each equation. Check your solution.

   a. \( 20 = 4x \)  
   c. \( -9t = -72 \)  
   e. \( 45 = 5p \)  
   g. \( -5r = 25 \)

   b. \( -7y = 49 \)  
   d. \( 3y = -21 \)  
   f. \( 72 = 12q \)  
   h. \( -12n = 60 \)
3. A pair of shoes costs 500 Birr. This is 280 Birr less than the cost of a pair of jeans. Find the cost of the jeans.

4. The average lifespan of a tiger is 22 years. This is 13 years less than a lion. Write and solve an equation to find the lifespan of a lion.

5. Bemnet wants to buy books that cost 300 Birr. If she saves 20 Birr each week, in how many weeks will she have enough money for the books?

**Activity 4.5**

In grade seven, Yoseph gets a reward of 2 books for each math test he scored full marks, and 3 books for passing to the next grade.

Yoseph has 9 books. The model illustrates the equation $2x + 3 = 9$, where $x$ represents the number of tests he scored all marks.

To solve $2x + 3 = 9$, remove three 1-tiles from each side of the box.

Then divide the remaining tiles into two equal groups. The solution of $2x + 3 = 9$ is 3.

Solve each equation by using models or a drawing.

a. $3x + 2 = 8$

b. $7 = 4x + 3$

A two-step equation has two different operations. In the equation $2x + 3 = 9$, $x$ is first multiplied by 2 and then 3 is added. So, to solve such a two-step equation, you should “undo” the addition operation first and then the multiplication.

**Note:**

Steps to solve a two-step equation like $3x + 4 = 16$ or $2x - 1 = -3$:

**Step 1:** Undo the addition or subtraction first.

**Step 2:** Then undo the multiplication or division.
UNIT 4: LINEAR EQUATIONS

Example 4.15

Solve: \(2x + 3 = 9\). Check your solution.

Solution:

\[
\begin{align*}
2x + 3 &= 9 & \text{Write the equation.} \\
2x + 3 - 3 &= 9 - 3 & \text{Undo the addition first by subtracting 3 from each side.} \\
\frac{2x}{2} &= \frac{6}{2} & \text{Next, undo the multiplication by dividing each side by 2.} \\
x &= 3 & \text{Simplify.}
\end{align*}
\]

Check the solution. Since \(2(3) + 3 = 9\), the solution is 3.

Example 4.16

Solve: \(-5y - 17 = 3\).

Solution:

\[
\begin{align*}
-5y - 17 &= 3 & \text{Write the equation.} \\
-5y - 17 + 17 &= 3 + 17 & \text{Undo the subtraction first by adding 17 to each side.} \\
-5y &= 20 & \text{Simplify.} \\
\frac{-5y}{-5} &= \frac{20}{-5} & \text{Next, undo the multiplication by dividing each side by \(-5\).} \\
y &= -4
\end{align*}
\]

Therefore, the solution is -4.

Example 4.17

Solve: \(25 = \frac{1}{4}n - 3\).

Solution:

\[
\begin{align*}
\frac{1}{4}n - 3 &= 25 & \text{Write the equation.} \\
\frac{1}{4}n - 3 + 3 &= 25 + 3 & \text{Add 3 to each side.} \\
\frac{1}{4}n &= 28 & \text{Simplify.}
\end{align*}
\]
\[ 4 \times \frac{1}{4} n = 4 \times 28 \]
Multiply each side by 4.

\[ n = 112 \]

Therefore, the solution is 112.

**Example 4.18**

Your class needs 600 Birr for building simple Biogas Digester Plant. Since the school provides them only 210 Birr, they decide to raise the rest by selling donuts for a profit of 1.50 Birr per donut. How many donuts will they need to sell?

**Solution:**

Amount provided by the school plus 1.50 per donut sold equals 600 Birr.

Let \( d \) represent the number of donuts.

Then, the equation is:

\[ 210 + 1.50d = 600 \]

So that:

\[ 210 - 210 + 1.50d = 600 - 210 \]

\[ 1.50d = 390 \]

Subtract 210 from each side.

\[ 1.50 \frac{d}{1.50} = \frac{390}{1.50} \]

Simplify.

\[ d = 260 \]

Divide each side by 1.50.

**Check:**

\[ 210 + 1.50d = 600 \]

Write the original equation.

\[ 210 + 1.50 \ (260) \overset{?}{=} 600 \]

Replace \( d \) with 260.

\[ 210 + 390 \overset{?}{=} 600 \]

Simplify.

\[ 600 = 600 \checkmark \]

The sentence is true.

Therefore, they need to sell 260 donut.

**Example 4.19**

The sum of the ages of a man and his wife is 96 years. The man is 6 years older than his wife. How old is his wife?

**Solution:**

Let \( m \) be the age of the man and \( w \) be the age of his wife:
UNIT 4: LINEAR EQUATIONS

Then \( m+w=96 \), and \( m=w+6 \)

\[
6+w+w=96 \quad \text{By substituting}
\]
\[
6+2w = 96-6 \quad \text{Subtracting 6 from both sides}
\]
\[
2w+6-6 = 96-6 \quad \text{Dividing both sides by 2}
\]
\[
2w = 90 \quad \text{Dividing both sides by 2}
\]
\[
\frac{2w}{2} = \frac{90}{2}
\]
\[
w = 45
\]

Therefore, the age of his wife is 45 years and the man is 51 years old.

Exercise 4.5

1. Solve each equation. Check your solution.
   
   a. \( 2x + 4 = 10 \)
   
   b. \( 3x + 5 = 17 \)
   
   c. \( 71 = 15 + 8x \)
   
   d. \( 4h - 6 = 22 \)
   
   e. \( 4 + 5r = -11 \)
   
   f. \( -3n - 8 = 7 \)
   
   g. \( -6r + 1 = -17 \)
   
   h. \( -3y - 5 = 10 \)
   
   i. \( -3n - 8 = 7 \)

2. Solve each equation. Check your solution.
   
   a. \( -1 = 1/2 \ a + 9 \)
   
   b. \( 10 - 2/3p = 52 \)
   
   c. \( n/-3 - 2 = -18 \)
   
   d. \( 2r - 3.1 = 1.7 \)
   
   e. \( 4t + 3.5 = 12.5 \)
   
   f. \( 16b - 6.5 = 9.5 \)
   
   g. \( 5w + 9.2 = 19.7 \)
   
   h. \( 16 = 0.5r - 8 \)
   
   i. \( 0.2n + 3 = 8.6 \)

3. Sosina wants to buy some drawing pencils, each costing 14 birr, and a drawing book that cost 23 birr. She has 65 birr to spend. Write and solve an equation to find how many pencils she can buy.

4. Suppose the current temperature in Debre Markos is 54°F. It is expected to rise 2°F each hour for the next several hours. In how many hours will the temperature be 78°F?

5. The perimeter of a rectangle is 40 cm. The width is 8 cm shorter than the length. Write and solve an equation to find the dimensions of the rectangle.
4.2.1 Linear Equations Involving Brackets

Activity 4.6

Compare each of the following pairs of equations. How they are related?

a. \(2(x + 1) - 3(1 - x) = 4\) and \(5x - 1 = 4\)

b. \(7(1 - 2x) + 12(x - 1) = 3\) and \(2x + 5 = -3\)

In this section, you will learn about how to solve linear equations containing one or more brackets. To solve an equation containing brackets such as \(5(2x + 3) = 25 - (x + 10)\), transform it into an equivalent equation that does not have brackets.

Example 4.20

Solve: \(5(x + 2) - 6(1 - x) = -18\).

Solution:

\[
\begin{align*}
5(x + 2) - 6(1 - x) &= -18 \\
5x + 10 - 6 + 6x &= -18 \\
5x + 6x + 10 - 6 &= -18 \\
11x + 4 &= -18 \\
11x + 4 - 4 &= -18 - 4 \\
11x &= -22 \\
\frac{11x}{11} &= \frac{-22}{11} \\
x &= -2
\end{align*}
\]

Write the equation.
Removing brackets.
Collecting like terms together.
Simplifying.
Subtracting 4 from both sides.
Dividing both sides by 11.

Check:

\[
\begin{align*}
5(x+2)-6(1-x) &= -18. \\
5\times(-2+2)-6\times(1-(-2)) &= -18. \\
-18 &= -18 \text{ is true.}
\end{align*}
\]

Therefore, \(x = -2\) is the solution.

Example 4.21

Hikma thinks of a number and adds 9 to it. She then multiplies her answer by 3 and gets 66. What was her original number?
Solution:

Let Hikma’s original number be \(x\).

Hikma added 9 to her number, which would give \(x + 9\).

Then she multiplied this by 3, which would give \(3(x + 9)\).

Her answer was 66, so we now know that \(3(x + 9)\) is equal to 66:

\[
3(x + 9) = 66 \quad \text{Write the equation.}
\]
\[
3x + 27 = 66 \quad \text{Multiply out the brackets.}
\]
\[
3x + 27 - 27 = 66 - 27 \quad \text{Subtract 28 from both sides.}
\]
\[
3x = 39
\]
\[
x = 13 \quad \text{Divide both sides by 4.}
\]

So, Hikma’s original number was 13.

Adding the same quantity to each side of an equation; subtracting the same quantity from each side of the equation; and multiplying and dividing both sides of the equation by the same non-zero quantity results an equivalent equation.

Definition:

Two equations are said to be equivalent if and only if they have exactly the same values for the variables that satisfy each of the equations.

Example 4.22

Show that: \(-3(x - 5) + 1 = 4 + x\) and \(\frac{16-4x}{2} = 4 + x\) are equivalent equations.

Solution:

First equation: \(-3(x - 5) + 1 = 4 + x\) \quad \text{Write the equation.}

\[
-3x + 15 + 1 = 4 + x \quad \text{Use distributive property.}
\]
\[
-3x + 16 = 4 + x
\]
\[
-3x - x + 16 = 4 + x - x \quad \text{Subtract } x \text{ from both sides.}
\]
\[
-4x + 16 - 16 = 4 - 16 \quad \text{Subtract 16 from both sides.}
\]
\[
-4x = -12
\]
\[
\frac{-4x}{-4} = \frac{-12}{-4} \quad \text{Divide both sides by } -4.
\]
\[
x = 3
\]
Second equation: \( \frac{26 - 4x}{2} = 4 + x \)

\[
\left( \frac{26 - 4x}{2} \right) \times 2 = (4 + x) \times 2
\]

Multiply both sides by 2.

26-4x = 8+2x.

26-8 = 2x+4x

18 = 6x

18/6 = 6x/6

3 = x

Collecting like terms together.
Dividing both sides by 6.

Since they have the same solution, \(-3(x - 5) + 1 = 4 + x\) and \(\frac{16 - 4x}{2} = 4 + x\) are equivalent equations.

**Exercise 4.6**

1. Solve each equation.
   a. 2x-45=7x
   b. (2x+3)=3(x+8)
   c. (5x-7)=2(3x+7)
   d. 24x-28=8x+20
   e. 2(3x+4)=6-2x-5
   f. 4x-(3x-5)=40

2. Identify each of the following pairs of equations as equivalent or not equivalent.
   a. 21x=42 and 3x=6
   b. 2x+(-6)=14 and 2x=14+6
   c. 2x+8=18 and 2x=18-12
   d. \(\frac{3}{5}x - \frac{3}{7} = 10\) and 21x - 365 = 0

3. Solve for x
   a. ax+b=cx+d, where a\(\neq\)c
   b. x+y=b(y-x), where b\(\neq\)-1

4. In the figure to the right, write an equation to represent the length of AB, and find the value of x.
Problem Solving

1. Ten years from now Denkie’s age will be 3 times her present age. Find Denkie’s present age.

2. In a school with grade 7 and 8 only, the number of grade 7 students is twice as the number of grade 8 students. If the total number of students at the school is 3,000, how many grade 7 students are there? How many grade 8 students are there?

4.2.2 Linear Equations Involving Fractions

Activity 4.7

Suppose the following two balances are in equivalent condition; can you determine the missing number of oranges in the second balance?

Some equations contain fractional coefficients. A good approach is to multiply the whole equation by a natural number, to clear the fractions away. You would normally use the lowest common denominator of the fractions. The solution for a real life problem will not necessarily be an integer. The operation rules working in the integers also works for fractions.

Let \( \frac{a}{b} \) and \( \frac{c}{d} \) be fractions such that, \( a,d \neq 0 \). Then \( \frac{a}{b}x + \frac{c}{d} = 0 \) is a linear equation involving fractions.

Example 4.23

Solve:

a. \( \frac{1}{2}x + 3 = \frac{21}{5} \)

b. \( 3.2x + 6.4 = 12.9 \)

c. \( \frac{a}{b}x + \frac{c}{d} = 0 \)

Solution:

a. \( \frac{1}{2}x + 3 = \frac{21}{5} \)  
Write the equation.

\[ \frac{1}{2}x + 3 - 3 = \frac{21}{5} - 3 \]  
Subtract 3 from both sides.

\[ \frac{1}{2}x = \frac{21 - 15}{5} = \frac{6}{5} \]  
Simplify.
\[
2 \times \frac{1}{2} \times \frac{x}{5} = \frac{6}{5} \times 2 \\
\frac{x}{5} = \frac{12}{5} \\
x = \frac{12}{5} 
\]

b. \(3.2x + 6.4 = 12.9\)

\[3.2x + 6.4 - 6.4 = 12.9 - 6.4 \quad \text{why?}\]

\[2x = 6.5\]

\[x = \frac{6.5}{3.2} = 2.03125\]

c. \(\frac{a}{b}x + \frac{c}{d} = 0\)

\[
\frac{a}{b}x + \frac{c}{d} - \frac{c}{d} = 0 - \frac{c}{d} \\
\frac{a}{b}x = -\frac{c}{d} \\
\left(\frac{b}{a}\right) \times \left(\frac{a}{b}x\right) = \left(-\frac{c}{d}\right) \times \left(\frac{b}{a}\right) \\
x = \frac{-bc}{ad} 
\]

**Example 4.24**

Aminat thinks of a number and subtracts \(\frac{5}{2}\) from it. She multiplies the result by 8. Finally she obtains a number which is 3 times the number she thought. What was the number?

**Solution:**

Let the number be \(x\),

According to the question, \((x - \frac{5}{2}) \times 8 = 3x\)

\[8x - \frac{40}{2} = 3x\]

\[8x - 3x = \frac{40}{2}\]  \(\text{(Why?)}\)

\[5x = 20\]

\[x = 4\]

Thus, the number is 4.
Example 4.25

Solve: \( \frac{5y+8}{6} - \frac{3y+2}{4} = 1 \)

**Solution:**

The LCM of 6 and 4 is 12, so multiply both sides of the equation by 12.

\[
12 \times \left( \frac{5y+8}{6} - \frac{3y+2}{4} \right) = 12 \times 1
\]

\[
12 \times \left( \frac{5y+8}{6} \right) - 12 \times \left( \frac{3y+2}{4} \right) = 12 \times 1 \quad \text{Why?}
\]

\[
2(5y+8) - 3(3y+2) = 12 \quad \text{Distributive property of multiplication}
\]

\[
10y + 16 - 9y - 6 = 12 \quad \text{Why?}
\]

\[
y + 10 = 12 \quad \text{Why?}
\]

\[
y = 2
\]

---

**Exercise 4.7**

1. Solve the following equations
   
   a. \( h + \frac{1}{7} = 19 \)
   b. \( 7x + 2 = 3x + 12 \)
   c. \( 14x - 9 = 2x + 18 \)
   d. \( \frac{3x - 1}{7} = \frac{9 - x}{2} \)
   e. \( \frac{7x - 5}{10} = \frac{x}{2} \)
   f. \( \frac{x - 1}{2} = \frac{x + 1}{3} \)

2. Find the value of \( x \) that satisfy each of the following equations:
   
   a. \( 4(x - 12) = 2x + 10 \)
   b. \( 2(3x + 5) = 6x + 11 \)
   c. \( 4(7x - 21) = 14(7x - 21) \)
   d. \( 3(x + 1) + 1 = 3x + 4 \)

3. A taxi driver has fixed charge 50 Birr and then charges 10 Birr per kilometer.
   
   a. Construct a conversion table to enable you to estimate the cost of the following taxi rides for 5km and 8.5km
   
   b. If a trip cost 80 Birr, for how much km did the customer used the taxi?

4. In some trip a dollar had an exchange rate of 32.34 Birr.
   
   a. Construct a conversion table for 1, 2, 3, 4, 10, 50 dollars into Birr
   
   b. From the table estimate how many dollars you would get for 1500 Birr.
Problem Solving

1. Zekarias thinks of a number. When he multiplies his number by 5 and subtracts 16 from the result he gets the same answer as when he adds 10 to his number and multiplies that result by 3. Find the number.

2. The denominator of a number is greater than its numerator by 8. If the numerator is increased by 17 and the denominator is decreased by 1, the number obtained is $\frac{3}{2}$. Find the rational number.

4.3 Cartesian Coordinate System

Mathematicians use a pair of perpendicular number lines, to form coordinate system and locate specific points on a plane. This concept of locating points on a coordinate system is applicable in different life experiences. For example, in geography it is applied in locating and specifying places on a map.

4.3.1 The Four Quadrants of the Cartesian Coordinate Plane

Activity 4.8

Specify the indicated points A, B, C, D, and E in terms of the vertical line (line of longitude) and horizontal line (line of latitude) as shown on the map of Ethiopia below. For Example, point F is located at (45° E, 10° N).

A Cartesian coordinate plane is formed when two number lines which are perpendicular to each other intersect at their zero points. The intersection point of the two lines is the
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origin and has coordinates (0, 0). The horizontal line is the x-axis, and the vertical line is the y-axis.

The point (0, 0) is an example of an **ordered pair**. The numbers in an ordered pair are called **coordinates**. The coordinates give the location of the point.

**Example 4.26**

Refer the diagram to the right and answer the following questions.

a. Which point is at (-2, 2)?

b. What are the coordinates of B, K, F, I or D?

c. Which point has the same x and y coordinate?

**Solution:**

a. (-2,2) is the coordinate of C

b. B(2,3), K(-1,3), F(1,-2), I(3,-2), D(3,0)

c. A(1,1) and G(0,0) have the same x and y coordinate.

The location of any point P on a coordinate plane can be described by an ordered pair of numbers (a, b), where a perpendicular from P to the x-axis intersects at a point with coordinate a, and a perpendicular from P to the y-axis intersects at a point with coordinate b. The point is denoted as P(a, b). In P(a, b) the first coordinate ‘a’ is called an **abscissa** and the second coordinate ‘b’ is called **ordinate**.

The x-axis and y-axis separate the plane into four regions called quadrants; numbered counter clockwise as quadrant-I, quadrant-II, quadrant-III and quadrant-IV as shown in the figure below.

![Coordinate System Diagram](image-url)
Example 4.27

Mark the following coordinate points on a rectangular coordinate system and state in which quadrant each coordinate point is located.

a. (4,3)  
   b. (-2,4)  
   c. (-6,-5)  
   d. (3,-5)

Solution:

The location of the points is indicated as in the figure.

a. (4, 3) is in quadrant I,  
   b. (-2,4) is in quadrant II,  
   c. (-6,-5) is in quadrant III and  
   d. (3,-5) is in quadrant IV,

Note:

1. If P(a, b) is a point in 1st quadrant, then both abscissa (a) and ordinate (b) are positive.
2. If P(a, b) is a point in 2nd quadrant, then abscissa (a) is negative and ordinate (b) is positive.
3. If P(a, b) is a point in 3rd quadrant, then both abscissa (a) and ordinate (b) are negative.
4. If P(a, b) is a point in 4th quadrant, then abscissa (a) is positive and ordinate (b) is negative.

This method of locating and describing points is called the **rectangular (or Cartesian) coordinate system**. The name **Cartesian** is in honor of René Descartes, the French mathematician and philosopher who first used this system for graphing geometric figures.

Exercise 4.8

1. Using the diagram below, write coordinates of the points A, B, C, D and E
2. Answer the following questions based on the figure given to the right.
   a. Which point is at (-3, 1)?
   b. What are the coordinates of E?
   c. Which point has the same x and y coordinates?
   d. What are the coordinates of H?
   e. Which point has the largest y coordinate?
   f. Which point has the smallest x coordinate?

### 4.3.2 Coordinates and Graphs of Equations

So far you have learnt about plotting points on the coordinate system. A point is represented with pair of numbers, the first number indicates the value of x (abscissa) and the second number indicates the value of y (ordinate). Next you will see about graphs of linear equations.

**Activity 4.9**

1. Take any three points on the x-axis and determine their ordinate. Take any three points on the y-axis and determine their abscissa.

2. What can you say about set of points whose abscissas are uniformly 0 and whose ordinates are arbitrarily integers? How can this be related with the y-axis?

3. What can you say about set of points whose ordinates are uniformly 0 and whose abscissas are arbitrarily integers? How can this be related with the x-axis?

As you have observed from the activity above, every point on the x-axis has a y-coordinate of zero. Hence, the x-axis can be described as the set of all points (x, y) such that y = 0. Thus y = 0 is the equation of the x-axis.

Similarly, every point on the y-axis has an x-coordinate of zero. Hence, y-axis can be described as the set of all points (x, y) such that x = 0. Thus, x = 0 is the equation of the y-axis.
Example 4.28

Which points are on the x-axis? Which of them are on the y-axis?

a. (0,3)       c. (0,-5)       e. (0,0)
b. (-2,0)       d. (0,-1)       f. (2,0)

Solution:

Points A, C, D and E are points on the y-axis, because their abscissa is 0; points B, E and F are points on the x-axis, because their ordinate is 0.

Note:

- $x = 0$ is the equation of the y-axis.
- $y = 0$ is the equation of the x-axis.

Activity 4.10

1. Find at least 9 points whose abscissa is 2 and whose ordinates are between -5 and 5.
2. Find at least 9 points whose ordinate is 3 and whose abscissas are between -5 and 5.
3. Plot the points obtained in (1) and (2) on the coordinate plane.
4. Draw a line passing through these points.
5. What can you say about the equations represented by points obtained in (1) and (2)?

Look at the graphs below. The graph of the equation $x = a$ is the vertical line perpendicular to the x-axis through the point with coordinates. Similarly, the graph of the equation $y = b$ is the horizontal line perpendicular to the y-axis through the point with coordinates.
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Example 4.29

Sketch the graph of each of the following

a. $x = -2$

b. $y = -3$

Solution:

a. The equation $x = -2$ represents all points $(x, y)$ for which $x = -2$ and $y$ is any rational number.

<table>
<thead>
<tr>
<th>$x = -2$</th>
<th>x</th>
<th>-2</th>
<th>-2</th>
<th>-2</th>
<th>-2</th>
<th>-2</th>
<th>-2</th>
<th>-2</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>-4</td>
<td>-3</td>
<td>-2</td>
<td>-1</td>
<td>0</td>
<td>1</td>
<td>2</td>
<td>3</td>
</tr>
</tbody>
</table>

b. The equation $y = -3$ represents all points $(x, y)$ for which $y = -3$ and $x$ is any rational number.

<table>
<thead>
<tr>
<th>$y = -3$</th>
<th>x</th>
<th>-4</th>
<th>-3</th>
<th>-2</th>
<th>-1</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>-3</td>
<td>-3</td>
<td>-3</td>
<td>-3</td>
<td>-3</td>
<td>-3</td>
<td>-3</td>
<td>-3</td>
<td>-3</td>
<td>-3</td>
</tr>
</tbody>
</table>

Activity 4.11

a. Complete the following table

<table>
<thead>
<tr>
<th>x</th>
<th>-4</th>
<th>-3</th>
<th>-2</th>
<th>-1</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y = 2x$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

b. Plot the points on Cartesian coordinate plane.

c. Join the points using straight line.

d. What does the graph of $y = 2x$ look like?
From the activity, you observed that both the x-values and y-values of the sample points vary. Hence, the graph of the equation is not a vertical or horizontal line. It represents the set of all points (x, y), where x and y are related by the equation \( y = 2x \).

**Example 4.30**

Draw the graph of \( y = -3x \)

**Solution:**

**Step 1:** Choose some values for x, for example let \( x = -2, -1, 0, 1, 2 \)

**Step 2:** Find the value of y for each x in the table.

<table>
<thead>
<tr>
<th>x</th>
<th>-2</th>
<th>-1</th>
<th>0</th>
<th>1</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>6</td>
<td>3</td>
<td>0</td>
<td>-3</td>
<td>-6</td>
</tr>
</tbody>
</table>

**Step 3:** Plot the points \((-2, 6), (-1, 3), (0, 0), (1, -3), (2, -6)\) and join them by a straight edge to get a straight line.

**Step 4:** Label the line as \( y = -3x \).

**Example 4.31**

Draw the graph of the following equations on the same coordinate plane:

\[
y = x, \quad y = 2x, \quad y = \frac{1}{2}x
\]

**Solution:**

Determine the coordinates of the equations.

<table>
<thead>
<tr>
<th>x</th>
<th>-3</th>
<th>-2</th>
<th>-1</th>
<th>0</th>
<th>1</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y = x )</td>
<td>-3</td>
<td>-2</td>
<td>-1</td>
<td>0</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>( y = 2x )</td>
<td>-6</td>
<td>-4</td>
<td>-2</td>
<td>0</td>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td>( y = \frac{1}{2}x )</td>
<td>(-)</td>
<td>(-1)</td>
<td>(-\frac{1}{2})</td>
<td>0</td>
<td>(\frac{1}{2})</td>
<td>1</td>
</tr>
</tbody>
</table>
Draw graphs of the equations.

Example 4.32

Draw the graph of the following equations on the same coordinate plane:

\[ y = -x, \quad y = -2x, \quad y = -\frac{1}{2}x \]

Solution:

Determine the coordinates of the equations.

<table>
<thead>
<tr>
<th></th>
<th>-3</th>
<th>-2</th>
<th>-1</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y = -x )</td>
<td>3</td>
<td>2</td>
<td>1</td>
<td>0</td>
<td>-1</td>
<td>-2</td>
<td>-3</td>
</tr>
<tr>
<td>( y = -2x )</td>
<td>6</td>
<td>4</td>
<td>2</td>
<td>0</td>
<td>-2</td>
<td>-4</td>
<td>-6</td>
</tr>
<tr>
<td>( y = -\frac{1}{2}x )</td>
<td>( \frac{3}{2} )</td>
<td>1</td>
<td>( \frac{1}{2} )</td>
<td>0</td>
<td>( -\frac{1}{2} )</td>
<td>-1</td>
<td>( \frac{3}{2} )</td>
</tr>
</tbody>
</table>

Draw graphs of the equations.
UNIT 4: LINEAR EQUATIONS

Note: The graphs of the lines shown in the two figures above, have equations of the form, where \( m \) takes the values \( \frac{1}{2}, 1 \) and 2, (example 4.23); -2, -1 and \( \frac{1}{2} \), example 4.24).

In the equation \( y = mx \), the number \( m \) is a measure of steepness and is called the slope of the line. The graph goes up from left to right (increases), when slope (\( m \)) is positive; and it goes down from left to right (decreases), when slope is negative.

Exercise 4.9

1. Answer the following.
   a. On which axis does the point A(0,6) lie?
   b. On which quadrant does the point B(-3, -6) lie?

2. Find the coordinates of two other points which are collinear (on the same line) with each of the following pairs of given points:
   a. P(2, 0), Q(3, 0)  
   b. P(0, 0), Q(0, 2)
   c. P(-2, 1), Q(-2, 2)  
   d. P(2, 2), Q(3,3)

3. What is the coordinates of the point of intersection of the line \( x = a \) and \( y = b \)?

4. Draw the graph of each of the equations.
   a. \( x = 3 \)  
   b. \( y = -1 \)  
   c. \( y = 3x \)  
   d. \( y = -4x \)

Activity 4.12

A ladder is used to reach upper levels of houses.

1. The rate of change of the ladder compares the height it is raised to the distance of its base from the house.

Write this rate as a fraction in simplest form.

2. Find the rate of change of a ladder that has been raised 100 cm and whose base is 50 cm from the building.

The term slope is used to describe the steepness of a straight line. The slope or rate of change or the steepness is the ratio of the rise, or vertical change, to the run, or horizontal
change. In linear equations, no matter which two points you choose, the slope of the line is always constant.

\[
\text{slope} = \frac{\text{rise}}{\text{run}}
\]

**Example 4.33**

Find the slope of the line.

**Solution:**

Choose two points on the line. The vertical change is 2 units while the horizontal change is 3 units.

\[
\text{slope} = \frac{\text{rise}}{\text{run}} = \frac{2}{3}
\]

The slope of the line is \(\frac{2}{3}\).

For any two points on a line, slope can be found by finding the ratio of the change in \(y\)-coordinates (rise) to the corresponding change in \(x\)-coordinates (run).

The difference \(x_2 - x_1\) is the run (horizontal difference), and the difference \(y_2 - y_1\) is the rise (vertical difference).

Therefore, slope of the line is expressed as:

\[
m = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1}
\]
**Definition:**

Given two points A\((x_1, y_1)\) and B\((x_2, y_2)\) with \(x_1 \neq x_2\), the slope \(m\) of the line \(AB\) passing through these points is given by:

\[
m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{y_1 - y_2}{x_1 - x_2}
\]

Thus, for any arbitrary point \(p(x, y)\) on the line, the equation of the line is given by

\[
y - y_1 = m(x - x_1)
\]

**Example 4.34**

Find the slope of the line that passes through the pair of points C\((-1, -4)\), and D\((2, 2)\).

**Solution:**

Let \((x_1, y_1) = (-1, -4)\), and \((x_2, y_2) = (2, 2)\).

\[
m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2 - (-4)}{2 - (-1)} = \frac{6}{3} = 2
\]

**Example 4.35**

Given pairs of points A\((3,2)\) and B\((5,4)\),

- a. Find the slope of the line AB through the two points.
- b. Find the equation of the line.
- c. Find any other point on the line.

**Solution:**

Let \((x_1, y_1) = (3, 2)\) and \((x_2, y_2) = (5,4)\). Since slope of the line is given by,

\[
m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4 - 2}{5 - 3} = \frac{2}{2} = 1
\]
Hence the equation of the line becomes

\[
\frac{y - y_1}{x - x_1} = \frac{y - 2}{x - 3} = 1
\]

\[y - 2 = x - 3\]

\[y = x - 1\]

Therefore, \( y = x - 1 \) is the equation of the line.

To find another point on the line let us take any value of \( x \) and find \( y \) according to the equation. That is, take \( x = 1 \), then \( y = 0 \). Thus, \( (1,0) \) is a point on the line.

**Example 4.36**

Given the pairs of points \( (x_1, y_1) \) and \( (x_2, y_2) \).

a. Find the slope of the line \( PQ \) through the two points.

b. Find the equation of the line.

c. Find another point on the line.

**Solution:**

Let \( (x_1, y_1) = (-3, 4) \) and \( (x_2, y_2) = (-1, 0) \). Then,

\[
m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{0 - 4}{-1 - (-3)} = \frac{-4}{2} = -2
\]

Hence, the equation of the line becomes

\[
\frac{y - y_1}{x - x_1} = \frac{y - 0}{x - (-1)} = -2
\]

Therefore, \( y = -2x - 2 \) is the equation of the line.

To find another point on the line let’s take any value of \( x \) and find \( y \) according to the rule. That is, take \( x = 1 \), then \( y = -4 \). Thus, \( (1, -4) \) is a point on the line.

**Exercise 4.10**

1. Find the slope of each line.

   a. 
   
   ![Diagram a]

   b. 
   
   ![Diagram b]

   c. 
   
   ![Diagram c]
2. Find the slope of the line that passes through each pair of points.
   a. A(4, 4) and B(7, 5)  
   b. B(-2, 1) and D(0, -3)  
   c. C(-6, -3) and K(-2, -1)  
   d. D(-3, -2) and B(5, 4)  
   e. E(-4, 2) and D(1, 5)  
   f. F(-6, 5) and F(3, -3)

3. The points given in the table lie on a line. Find the slope of the line. Then graph the line.

<table>
<thead>
<tr>
<th>x</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>1</td>
<td>3</td>
<td>5</td>
<td>7</td>
</tr>
</tbody>
</table>

4. Find the slope of a road that rises 12 meters for every horizontal change of 100 meters.

5. Write the equation of the line that passes through each pair of points and has the indicated slope.
   a. (2, 10) and \( m = -4 \)  
   b. (4, -4) and \( m = 23 \)  
   c. (0, 0) and \( m = 53 \)

6. Write the equation of the line that passes through (0,0) and (2,6). Then graph the line.

### 4.4 Applications

Real life problems such as sale tax, sport activities, contract work and climate issues can be described by linear equations.

There are four-Step Problem-Solving Process/ procedures.

**Step 1: Understanding the problem:** State the problem in your own words and identify what is to be found or what is needed.

**Step 2: Devising a plan:** Look for a pattern and write appropriate mathematical equation.

**Step 3: Carrying out the plan:** Implement the strategy in step 2 and perform the necessary actions or computations.

**Step 4: Looking back:** Check the results in the original problem.
Example 4.37

A book is purchased for a total of Birr 48.15 including sales tax. If the tax rate is 7%, find the original price of the book before sales tax is added.

Solution:

Step 1: Understanding the problem

Given: Total cost=48.15 Birr and Tax rate=7%

Required: Cost before tax = x

Step 2: Devising a plan (Formulate mathematical equation)

Sales tax amount: 0.07x

Total Cost: x+0.07x = 48.15 Birr

1.07x = 48.15Birr

Step 3: Carrying out the plan (Solving)

1.07x = 48.15Birr

100(1.07x) = 100(48.15Birr).

107x = 4815 Birr

\[ x = \frac{4815Birr}{107} = 45 Birr \]

Therefore, the original price was 45 Birr

Step 4: Looking back (Check)

Sale with tax = 45 + 45 × \( \frac{7}{100} \) = 45 + 3.15 = 48.15 Birr

Example 4.38

For school-based sports competition event, a student pays 5 Birr per ticket and a non-student pays 13 Birr per ticket. If a certain number of students and 100 non-students buy tickets, find the number of students if the total revenue is 1900 Birr.

Solution:

Step 1: Given: Students pay 5Birr/Ticket, 100 nonstudents each pay 13Birr (Payment/Ticket), and total revenue 1900Birr

Required: Number of students; let \( x \) be the number of students

Step 2: Revenue collected from students (5x Birr) and from nonstudents (100 × 3 Birr) is equal to 1900 Birr (total);

Therefore, the equation is: 5x Birr + 1300 Birr = 1900 Birr
Step 3: \[5x \text{ Birr} = 1900 \text{ Birr} - 1300 \text{ Birr}.\]
\[5x \text{ Birr} = 600 \text{ Birr}\]
\[x = \frac{600 \text{ Birr}}{5 \text{ Birr}} = 120 \text{ student}\]

Therefore, the number of students who attained the event is 120.

Step 4: Total revenue \[= (5 \times 120) \text{ Birr} + (100 \times 13) \text{ Birr}\]
\[= 600 \text{ Birr} + 1300 \text{ Birr} = 1900 \text{ Birr}\]

Example 4.39

Three men took a 20,000 Birr project and if two of them got equal amount and one of the project coordinators got 2,000 Birr more than the others. Find the share of each.

Solution:

Step 1: Let \(x\) be the share of each of the two men, then the share of the coordinator is \((x+2000)\) Birr.

Step 2: The sum of shares of the two men and the coordinator is 20000 Birr

Thus, \((x+x+(x+2000))\) Birr = 20000 Birr

Step 3: \(x+x+(x+2000) = 20000.\)

\[3x+2,000 = 20,000.\]
\[3x = 18,000.\]
\[x = 6,000.\]

Therefore, the two men got 6000 Birr each, and the coordinator got 8000 Birr

Step 4: The sum of shares of the two men and the coordinator is 20000 Birr

Then, \((6000+6000+(6000+2000))\) Birr = 20000 Birr

Example 4.40

An observer can estimate the distance to an approaching thunderstorm by counting the seconds between a flash of lightning and the resulting sound of thunder. Every 5 seconds sound travels approximately 1 mile. If you count up to 10 seconds before hearing the thunder, the storm is approximately 2 miles away.

a. Construct an input-output table for the coordinate pairs corresponding to the times of 5, 10, 15, 20, and 25 seconds.

b. Graph the coordinate pairs from your input-output table and connect them with a line, what is the slope of this line?
Solution:

Step 1: Since every 5 seconds sound travels approximately one mile, the input out-put pair coordinate will be \((5n,n)\).

Step 2:

<table>
<thead>
<tr>
<th>Time in second ((5n))</th>
<th>5</th>
<th>10</th>
<th>15</th>
<th>20</th>
<th>25</th>
</tr>
</thead>
<tbody>
<tr>
<td>Distance in Mile ((n))</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
</tr>
</tbody>
</table>

Step-3: Plotting the in-put out-put pair will give the following graph.

Exercise 4.11

1. A bicycle is on sale for 2100 Birr. This is 30% off the regular price (Discount). What is the regular price of the bicycle?

2. In a class there are 48 students. The number of girls is 3 times the number of boys. How many boys and how many girls are there in the class?

3. A farmer has sheep and hens. The sheep and hens together have 100 heads and 356 legs. How many sheep and hens does the farmer have?

4. The cost of a bottle of perfume, 548.90 Birr, was determined from the cost of the bottle plus the cost of the perfume. If the perfume costs 14.10 Birr more than the bottle, how much does the bottle cost?
UNIT 4: LINEAR EQUATIONS

Unit summary

- Like terms (similar) terms differ only in their coefficients.
- An algebraic expression is combination of algebraic terms
- Linear equations in one variable have only one solution.
- For any integer a, b, and c;
  a. If a=b, then a ± c = b ± c and the vice versa.
  b. If a=b, then ac=bc and the vice versa
  c. If a=b, then \( \frac{a}{c} = \frac{b}{c}, c \neq 0 \), and the vice versa.
- If a = b and c = d, then a + c = b + d and a - c = b - d.
- Two equations are equivalent if one of them is a nonzero multiple of other equation.
- If P(a, b) is in 1st quadrant, then both abscissa (a) and ordinate (b) are positive.
- If P(a, b) is in 2nd quadrant, then abscissa (a) is negative and ordinate (b) is positive.
- If P(a, b) is in 3rd quadrant, then both abscissa (a) and ordinate (b) are negative.
- If P(a, b) is in 4th quadrant, then abscissa (a) is positive and ordinate (b) is negative.
- Equation of a line through A(x_1,y_1) and B(x_2,y_2) with \( x_1 \neq x_2 \) is given by
  \[ \frac{y-y_1}{y-y_2} = \frac{x-x_1}{x-x_2} \]
  which is also known as the two-point form of equation of the line.

Slope-intercept form of equation of the line: is \( y=mx+b \)

Review Exercises

1. Translate the following statements into equations
   a. Eight less than three times a number is 23
   b. Seventeen is seven more than twice a number
   c. The quotient of a number and four, decreased by one, is equal to 5
   d. Fifteen equals 3 more than three times a number
   e. If 10 is increased by a quotient of a number and 6, the result is 5
   f. The difference between 12 and twice a number is 18
2. Determine the slope of the line passing through the given pairs, if it exists.
   a. (3, 5) and (9, 5)               c. (2, 3) and (-1,5)
   b. (4, 3) and (4, 9)

3. Write 350 as the sum of four consecutive whole numbers?

4. Fantahun noticed that if he begin with his age, add 24, divide the result by 2, and then subtract 6, he get his age back. What is his age?

5. Solve each of the following equations.
   a. 5(2x+1)+7(2x+1)=84               b. 3(2x-6)=4(2x-6)

6. Abdu is 10 years older than Hailu. Three years ago Abdu was 3 times as old as Hailu. Find their age.

7. The sum of two consecutive integers is three times their difference. What is the larger number?

8. The sum of eight times a number and five times another number is 184. The difference of the second number from the first is -3. Find the numbers.

9. The perimeter of a rectangular field is 628m. The length of the field exceeds its width by 6m. Find the dimensions of the rectangle.

10. Suppose the current temperature is 54°F. It is expected to rise 2°F each hour for the next several hours. In how many hours will the temperature be 78°F?
UNIT - 5

PERIMETER AND AREA OF PLANE FIGURES

Learning Outcomes:

After completing this unit, you will be able to:

- Construct different quadrilaterals
- Describe properties of quadrilaterals
- Find the perimeter of triangles, parallelograms, trapeziums, rhombus, circles and composite shapes
- Derive formula for area of a triangle, quadrilaterals, circles and composite shapes.
- Calculate areas of triangle, quadrilaterals, circles and composite shapes.
- Apply the concept of area and perimeter of Plane figures in solving real life problems

Key Terms

- Triangle
- Quadrilateral
- Square
- Rectangle
- Parallelogram
- Trapezium
- Kite
- Circle
- Radius
- Diameter
- Circumference
- Diagonal
- Interior angle
- Exterior angle
- Adjacent angles
- Corresponding angles
Introduction

In this unit, you will broaden your knowledge of geometric figures. You will revise about the types of angles formed by intersecting lines. You will exercise how to construct plane figures and describe their properties using your construction as example. You will also learn more about triangles. Moreover, you will be able to calculate the area and perimeter of plane figures including triangles, quadrilaterals and circles.

5.1 Revision on Angles, Triangles and Polygons

Revision on Angles

Activity 5.1

In the figure below, a, b, c, d, e, f, g and h are angles formed when two parallel lines ℓ and m are crossed by a transversal line k. Copy the figure and measure all these angles using a protractor.

- a. Compare angles a and c, b and d, e and g, f and h.
- b. Compare angles d and f, c and e
- c. Compare a and g, b and h
- d. Compare a and c, b and f, e and g, d and h

The figure in the activity above shows two parallel lines and a transversal that crosses the parallel lines. These two parallel lines and the transversal line produces 8 angles at two intersecting points. Depending on their position, they are classified into three:

1. Alternate interior angles:

   The angles located here are called alternate interior angles. Alternate interior angles are equal.
   \[ m(\angle a) = m(\angle b) \]
2. Alternate exterior angles:

The angles located here are called alternate exterior angles. Alternate exterior angles are equal.

\[ m(\angle a) = m(\angle d) \text{ and } m(\angle b) = m(\angle c) \]

3. Corresponding angles:

The angles located here are called corresponding angles. Corresponding angles are equal.

\[ m(\angle a) = m(\angle b) \]

Angles on opposite sides of a vertex are equal. These angles are called **vertical opposite angles**. 

\[ m(\angle a) = m(\angle b) \]

**Note:**

Corresponding angles are sometimes (informally) known as F-angles, and Alternate interior angles are known as Z-angles, because of the resemblance to those letters.

Note that two parallel lines and a transversal line form: two pairs of alternate interior angles, two pairs of alternate exterior angles and four pairs of corresponding angles.

**Exercise 5.1**

In each of the figures, find the measures of the angles represented by the letters.
Revision on Triangles

Triangle is the most important figure we shall study in geometry. Triangle is a polygon of three sides. It is therefore the simplest type of polygon. The polygon \( \triangle ABC \) below represents a triangle and is denoted by \( \triangle ABC \).

**Theorem**

In any triangle, the sum of the measures of the interior angles is \( 180^\circ \).

\[
a + b + c = 180^\circ
\]

**Example 5.1**

Find the angles represented by letters in the figure below. Give a reason in each case.

**Solution:**

\( a = 65^\circ \) (Alternate interior to marked \( 65^\circ \) angle)

\( b = 74^\circ \) (Corresponding angle to marked \( 74^\circ \) angle)

Angles \( a \), \( b \), and \( c \) are the three interior angles of the triangle, so they add up to \( 180^\circ \).

Therefore \( c = 180^\circ - (a + b) \)

\[
= 180^\circ - (65^\circ + 74^\circ)
\]

\[
= 180^\circ - 139^\circ = 41^\circ
\]
Note:
An exterior angle of a triangle is equal to the sum of its two opposite non-adjacent interior angles.
\[ x = a + b \]

Types of Triangles

1. Classification of triangles **based on their angles**
   - Acute triangle (All 3 angles acute)
   - Right triangle (Contains 1 right angle)
   - Obtuse triangle (Contains 1 obtuse angle)

2. Classification of triangles **based on their sides**
   - Equilateral triangle (All 3 sides of equal length)
   - Isosceles triangle (at least 2 sides of equal length)
   - Scalene triangle (All 3 sides of different length)

**Example 5.2**

In the triangle shown below, find the value of \( x \) and then determine the size of the largest angle.

**Solution:**

Since the sum of angles of a triangle is 180°

\[
56° + x + 12° + x - 6° = 180°
\]
\[
62° + 2x = 180°
\]
\[
2x = 180° - 62°
\]
\[
2x = 118°
\]
\[
x = 59°
\]
Then, \( m(\angle L) = 56^\circ \)

\[ m(\angle M) = x + 12 = 59 + 12 = 71^\circ \]

\[ m(\angle N) = x - 6 = 59 - 6 = 53^\circ \]

Therefore, the largest angle is \( \angle M = 71^\circ \)

**Exercise 5.2**

1. Determine whether each of the following statement is true or false, and explain the reasons for your answer.
   
   a. Every equilateral triangle is an isosceles triangle.
   
   b. Some right triangles are isosceles triangles.
   
   c. Some isosceles triangles are scalene triangles.
   
   d. Some isosceles triangles are obtuse triangles.
   
   e. Some right triangles are scalene triangles.
   
   f. Some scalene triangles are acute triangles.

2. Find the value of \( y \) and the sizes of the angles of the triangle shown to the right.

3. A triangle has angles \( (x + 8)^\circ, (2x - 8)^\circ \) and \( 90^\circ \).
   
   a. Find the value of \( x \).
   
   b. Determine the sizes of the angles in the triangle.
   
   c. What kind of triangle is it?

4. In Figure to the right, if \( u \), \( y \) and \( x \) are degree measures of the angles marked, then what is the value of \( m(\angle u) + m(\angle y) \)?
5. Find the degree measure of the marked angle in the figure below.

6. In the Figure given to the right, $m(\angle ABC) = 32^\circ$, $m(\angle BHE) = 42^\circ$ and $m(\angle ADE) = 48^\circ$. Find $m(\angle NAD)$.

7. In the Figure given below, $DE \parallel AB$, $m(\angle BCA) = 108^\circ$, $m(\angle EDC) = 42^\circ$. Find $m(\angle B)$ and $m(\angle A)$.

8. In the figure below if $s$ and $t$ are parallel lines and the measure of $\angle f$ is $32^\circ$ and the measure of $\angle a$ is $40^\circ$, determine the measure of each of the unknown angles.

9. Given $AB \parallel CD$ and $m(\angle BAO) = 30^\circ$, $m(\angle DCO) = 40^\circ$. What is the measure of angle AOC?
Revision on Polygons

Polygons are classified according to the number of sides they have.

**Activity 5.2**

1. On a sheet of paper plot three points which are not on the same line (non collinear). Use straight edge and join the points by line segments. What type of figure have you got?

2. On the same way plot 4 points of which no three of them are on the same line. Join these points using line segments so that they form a closed plane figure.
   a. What type of figure have you got?
   b. How many sides and vertices does it have?

3. Continue what you did on number 2 above for 5 points, 6 points and so on and complete the following table.

<table>
<thead>
<tr>
<th>Number of points</th>
<th>Number of sides</th>
<th>Number of vertices</th>
<th>Number of interior angles</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>7</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>8</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

4. What can you conclude about the relationship between number of points used and the number of sides of a figure formed?

5. What is the name given to the plane figures you formed?

**Definition:**

a. A polygon is a simple closed plane figure formed by three or more line segments joined end to end. The line segments forming the polygons are called **sides** and the common end point of any two sides is called **vertex** (plural **vertices**) of the polygon.

b. A diagonal of a polygon is a line segment whose end points are non-consecutive vertices of the polygon.

c. A convex polygon is a simple polygon in which each of its interior angles measure less than 180°.

d. A concave polygon is a simple polygon which has at least one interior angle of measure greater than 180°.
The polygon shown to the right is a convex hexagon.

- **Sides** of the hexagon are: \( AB, BC, CD, DE, EF \) and \( FA \)

- **Angles** of the hexagon are: angles A, B, C, D, E, and F

- \( AE, AD \) and \( AC \) are **diagonals** of the polygon from one vertex

- A hexagon has 3 diagonals from one vertex, and these diagonals divide the hexagon into 4 triangles.

Therefore, the sum of the interior angle of the hexagon is equal to the sum of the interior angles of the four triangles.

That is, \( 4 \times 180° = 720° \)

**Definition:**

A polygon is said to be **regular polygon** if all its sides are equal and all its angles are also equal.

For example, squares and equilateral triangles are regular polygons but any rhombus is not a regular polygon.

**Activity 5.3**

1. Draw polygons whose names are given in the table and draw the diagonal(s) from one vertex of the polygon (if possible) and complete the table

<table>
<thead>
<tr>
<th>Number of sides</th>
<th>Name of the polygon</th>
<th>Number of diagonals formed</th>
<th>Number of triangles formed by the diagonal(s)</th>
<th>The sum of interior angles of the polygon</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>Triangle</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>Quadrilateral</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>Pentagon</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>Hexagon</td>
<td>3</td>
<td>4</td>
<td>720°</td>
</tr>
<tr>
<td>8</td>
<td>Heptagon</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>.</td>
<td>.</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

2. Can you generate a formula for number of diagonals from one vertex, number of triangles formed by these diagonals and the sum of all the interior angles of the n-sided polygon?
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**Note:**

For any \( n \)-sided polygon:

a. The number of diagonals from one vertex is \( n-3 \).

b. The number of triangles formed by the diagonals from one vertex is \( n-2 \).

c. The sum of all the interior angles is \((n-2) \times 180\,^\circ\).

**Example 5.3**

For an octagon, find

a. Number of diagonals from one vertex.

b. Number of triangles formed by the diagonals drawn from one vertex.

c. The sum of interior angles of the octagon.

**Solution:**

Octagon is 8 sided polygon, then

a. Number of diagonals from one vertex: \( n - 3 \), \( 8 - 3 = 5 \).

b. Number of triangles from one vertex: \( n - 2 \), \( 8 - 2 = 6 \).

c. The sum of interior angles of a polygon:

\[
(n - 2) \times 180^\circ, (8 - 2) \times 180^\circ = 1080^\circ
\]

**Exercise 5.3**

1. For a decagon (ten - sided polygon), find

   a. The number of diagonals drawn from one vertex.

   b. The number of triangles formed by the diagonal from one vertex.

   c. The sum of interior angles of the decagon.

2. For the given figure,

   a. Name the polygon.

   b. Name all the interior angles of the polygon.

   c. Draw the diagonals from all the vertices and name these diagonals.

   d. Find the sum of the interior angles of the polygon.
Problem Solving

Suppose three students stood on the three vertices of an equilateral triangle drawn on the ground. In how many different ways can they shake hands? What if they are four and stood on vertices of a square? What if they are five and stood on vertices of a pentagon? What if they are six and stood on vertices of a hexagon?

5.2 Four – sided Figures (Quadrilaterals)

Activity 5.4

Copy the triangle figure below, cut it (decompose) into the five shapes, and then form new shapes (compose) according to the following.

a. Join two of the shapes edge-to-edge to form a parallelogram.
b. Join three of the shapes edge-to-edge to form a square.
c. Join four of the shapes edge-to-edge to form a rectangle.
d. Join all five shapes edge-to-edge to form a square.
e. Join three shapes edge-to-edge to form a trapezium.

Note:
The sum of the interior angles of any quadrilateral is $360^\circ$.

Example 5.4

The figure below shows a quadrilateral. Find the value of $x$.

Solution:

Since the angles in a quadrilateral add to $360^\circ$

\[2x + x + 50 + 3x - 5 + 75 = 360^\circ\]

\[6x + 120 = 360^\circ\]
6x = 360° - 120°
6x = 240°

\[ x = \frac{240°}{6} \]

\[ x = 40° \]

**Special Quadrilaterals**

To perform geometric constructions; you need a *straight edge*, *protractor* and a *pair of compasses*. Using these basic tools you can construct a geometric figure with a better accuracy.

- **Straight edge**: A straight edge marked or unmarked, ruler is used to construct (draw) a line or a line segment through two given points.

- **Compass**: A compass is used to construct circles or arcs.

- **Protractor**: A protractor is used to measure the angles of the drawing.

**A. Parallelogram**

**Activity 5.5**

Draw two parallel lines; and again, draw another two parallel lines which intersect the first two parallel lines at A, B, C and D.

1. What type of quadrilateral is ABCD?
2. What can you say about the opposite sides of ABCD?
3. Can you list some properties of ABCD?
4. What is the name of this type of Quadrilateral?

**Definition:**

Parallelogram is a quadrilateral with pairs of opposite sides parallel.
Properties of Parallelogram

1. Opposite sides of a parallelogram are equal.
   \( AB = CD \) and \( AD = BC \).

2. Opposite sides of a parallelogram are parallel.
   \( \overline{AB} \parallel \overline{CD} \) and \( \overline{AD} \parallel \overline{BC} \).

3. Opposite angles of a parallelogram are equal.
   \( m(\angle A) = m(\angle C) \) and \( m(\angle B) = m(\angle D) \).

4. Consecutive angles of a parallelogram are supplementary.
   \( m(\angle A) + m(\angle B) = 180^\circ \), \( m(\angle B) + m(\angle C) = 180^\circ \)

5. The diagonals of a parallelogram bisect each other.
   The diagonals \( \overline{AC} \) and \( \overline{BD} \) intersect at \( O \) therefore, \( AO = CO \) and \( BO = DO \).

Example 5.5

Find lengths of \( AO \), \( OC \), \( BO \), and \( OD \) in parallelogram \( ABCD \).

Solution:

\[
AO = CO \quad \text{Diagonals of a parallelogram bisect each other.}
\]

\[
3y - 7 = 2x \quad \text{Equation - 1}
\]

\[
DO = BO \quad \text{Diagonals of a parallelogram bisect each other.}
\]

\[
x + 1 = y \quad \text{Equation - 2}
\]

\[
3(x + 1) - 7 = 2x \quad \text{Substitute equation 2 in equation 1}
\]

\[
3x + 3 - 7 = 2x
\]

\[
3x - 4 = 2x
\]

\[
3x - 2x = 4
\]

\[
x = 4 \text{ units}
\]

Thus, \( y = x + 1 \)

\[
y = 4 + 1 = 5 \text{ units}
\]

Therefore, \( AO = 3y - 7 = 3(5) - 7 = 15 - 7 = 8 \text{ units.} \)

\( OC = 2x = 2(4) = 8 \text{ units.} \)

\( BO = x + 1 = 4 + 1 = 5 \text{ units.} \)

\( DO = y = 5 \text{ units} \)

Hence \( AO = OC = 8 \text{ units} \) and \( BO = DO = 5 \text{ units} \).
Example 5.6

Find the measure of $\angle BAD$ in the given parallelogram.

**Solution:**

\[
m(\angle BAD) + 135^\circ = 180^\circ \text{ the sum of consecutive angles of a parallelogram} \]

\[
m(\angle BAD) = 180^\circ - 135^\circ \]

\[
m(\angle BAD) = 45^\circ \]

B. **Rectangle**

Activity 5.6

1. Construct a parallelogram $PQRS$ with $PQ = 4\text{ cm}$, $QR = 3\text{ cm}$ and $m(\angle SPQ) = 90^\circ$. (use ruler, protractor and a pair of compasses).

   a. Draw the diagonals.

   b. Measure the lengths of the diagonals.

   c. Measure and compare the lengths from each vertex to the intersections of the diagonals. What do you conclude?

2. What are the differences and similarities between a rectangle and a parallelogram?

Example 5.7

Construct rectangle $PQRS$ with $PQ = 6\text{ cm}$, $QR = 5\text{ cm}$ and $m(\angle P) = 90^\circ$ by using ruler, protractor and a pair of compasses.

**Solution:**

**Step 1:** Construct a line segment with length $6\text{ cm}$.

**Step 2:** Construct $m(\angle P) = 90^\circ$ and $m(\angle Q) = 90^\circ$.

**Step 3:** Mark point $R$ such that $QR = 5\text{ cm}$.

**Step 4:** Draw a line through $R$ and parallel to $\overline{PQ}$ so that it intersects with a line through $P$ and parallel to $\overline{QR}$. Let $S$ be the intersection point is point $S$.

$PQRS$ is the required rectangle.
**Definition:**
A rectangle is a parallelogram in which its angles are right angles.

**Properties of Rectangle**
1. A rectangle has all properties of a parallelogram.
2. All angles of a rectangle are right angles.
3. The diagonals of a rectangle are equal in length.

**Exercise 5.4**

1. In the figure to the right, ABCD is a parallelogram with $m(\angle ADC) = 47^\circ$. A line through A meets $\overline{CB}$ at E and $m(\angle AEB) = 68^\circ$. Find:
   a. $m(\angle ABC)$
   b. $m(\angle BAE)$
   c. $m(\angle BCD)$

2. In the parallelogram ABCD below, the given values are lengths of segments of the diagonals. What is the values of $x$ and $y$?

3. In the figure below, find the unknown marked angles.
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4. In the figures below, find the unknown or marked angles.
   a. 
   b. 

5. In a rectangle ABCD, the length of the diagonal $\overline{AC}$ is given by $(18x + 15)$ cm and the length of diagonal $\overline{BD}$ is given by $(20x + 11)$ cm. Find actual lengths of AC and BD.

6. Construct a rectangle EFGH with EF = 8 cm FG = 6 cm.

Problem Solving

The Ethiopian government declares the Green Legacy program. W/ro Keria and her family planted mango and avocado trees. She planted mango and avocado plants on two parallel rows. The distance between the rows is 5 m. Two mango trees and two avocado trees form a rectangular shape as shown in the figure below. If the diagonal length between the mango and avocado trees is 13 m, find the distance between two avocados and the length of the other diagonal.

C. Rhombus

Activity 5.7

a. Construct a parallelogram ABCD with $AB = BC = 5$ cm and $m(\angle DAB) = 70^\circ$.

b. Measure the lengths of the sides and angles of the figure you draw.

c. Compare side measures and also angle measures. What do you conclude? What do you call this special quadrilateral?

Definition:

A rhombus is a parallelogram in which two adjacent sides are equal.
Properties of a Rhombus

1. A rhombus has all properties of a parallelogram
2. All sides of a rhombus are equal.
3. The diagonals of a rhombus bisect each other at right angles.
4. The diagonals of a rhombus bisects the angles at the vertices.

Example 5.8

If ABCD is a rhombus with \( m(\angle ACB) = 40^\circ \), find \( m(\angle ADB) \)?

Solution:

\[
m(\angle ACB) = 40^\circ, \quad \text{Given} \\
m(\angle DAC) = 40^\circ, \quad \text{Alternate interior angles} \\
m(\angle ADB) + m(\angle DAC) = 90^\circ \quad \text{why?} \\
m(\angle ADB) + 40^\circ = 90^\circ \\
m(\angle ADB) = 90^\circ - 40^\circ \\
m(\angle ADB) = 50^\circ
\]

Example 5.9

The figure below ABCD is a rhombus with \( m(\angle ABC) = 124^\circ \) and \( BC = 12 \text{cm} \). Find:

a. The lengths AD, DC and AB

b. \( m(\angle DAB) \)

c. \( m(\angle BDC) \)

d. \( m(\angle AED) \)

Solution:

a. Since the figure is a rhombus all its sides are equal. One of its sides \( BC = 12 \text{cm} \) is given. Then, \( AB = BC = CD = AD = 12 \text{cm} \)

b. \( m(\angle DAB) + m(\angle ABC) = 180^\circ \quad \text{Consecutive angles of a parallelogram} \\
m(\angle DAB) + 124^\circ = 180^\circ \quad \text{Substitution} \\
m(\angle DAB) = 180^\circ - 124^\circ \\
m(\angle DAB) = 56^\circ \)
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c. \( m(\angle BDC) = \frac{1}{2} \left[ m(\angle ABC) \right] \) Diagonal of a rhombus is angle bisector

\[
m(\angle BDC) = \frac{1}{2} \left( 124^\circ \right)
\]

\[
m(\angle BDC) = 62^\circ
\]

d. \( m(\angle AED) = 90^\circ \) Diagonals of a rhombus form right angle at their intersection

D. Squares

Activity 5.8

1. Construct a rhombus ABCD with angle measure \( m(\angle ABC) = 90^\circ \) and AB = 4cm (use ruler, protractor and a pair of compasses).

2. What can you say about the figure you draw?

3. Define a square, and describe its properties by measuring or folding the figure?
   a. Draw diagonals and measure the lengths of the diagonals
   b. Measure and compare the lengths from each vertex to the intersections of the diagonals. What do you conclude?

4. What makes a square different from a rhombus?

Definition:

A square is a rectangle in which two of its adjacent sides are equal.

Properties of Square

1. A square has all properties of a rectangle and rhombus.

2. The diagonals of a square are equal and perpendicular bisectors of each other.

Example 5.10

ABCD is a square. \( \overline{CO} \) intersects \( \overline{DB} \) at E. If the measure of \( \angle DEC = 70^\circ \), then find the measure of \( \angle AOC \).
Solution:

Since each angle of a square is bisected by a diagonal
\[ m(\angle EBO) = \frac{1}{2} \left( 90^\circ \right) = 45^\circ \]
\[ m(\angle EBO) = m(\angle CED) = 70^\circ \quad \text{Vertical opposite angles} \]
\[ m(\angle BOE) + m(\angle OEB) + m(\angle EBO) = 180^\circ \quad \text{Sum of angles of a triangle} \]
\[ m(\angle BOE) + 70^\circ + 45^\circ = 180^\circ \quad \text{Substitution} \]
\[ m(\angle BOE) + 115^\circ = 180^\circ \]
\[ m(\angle BOE) = 180^\circ - 115^\circ = 65^\circ \]

Now \( m(\angle AOC) + m(\angle BOE) = 180^\circ \quad \text{Supplementary angles.} \)
\[ m(\angle AOC) + 65^\circ = 180^\circ \]
\[ m(\angle AOC) = 180^\circ - 65^\circ = 115^\circ \]

Thus, \( m(\angle AOC) = 115^\circ \)

Or, we can solve by
\[ m(\angle AOC) = m(\angle EBO) + m(\angle EBO) \quad \text{Measure of exterior angle is equal} \]
\[ = 70^\circ + 45^\circ \quad \text{to the sum of the two opposite} \]
\[ \text{interior angles} \]

Therefore, \( m(\angle AOC) = 115^\circ \)

Exercise 5.5

1. Find the length of the side of a rhombus whose diagonals are of lengths 6cm and 8cm.

2. In the Figure to the right, ABCD is a rhombus; with \( m(\angle BAD) = 135^\circ \). Find \( m(\angle ABD) \) and \( m(\angle ADC) \).

3. Figure PQRS is a rhombus. Point O is the bisector of diagonal QS and PR. If \( PO = x \), \( OR = y - 1 \), \( OQ = 2x + 7 \) and \( OS = 4y - 5 \), what is the value of \( x \) and \( y \)?
4. The size of the obtuse angle of a rhombus is twice the size of the acute angle of the rhombus. If its side length is 13 cm and the length of the shorter diagonal is 10 cm, then find:
   a. The sizes of the angles of the rhombus
   b. The length of the longer diagonal of the rhombus

5. WXYZ is a square. XV is a straight line. Find measure of \( \angle e \).

6. In the figure shown PRTU is a rectangle, and PQRS is a rhombus. Find \( m \) and \( n \).

7. In the figure to the right PQRS is a rectangle and PQT is an equilateral triangle. Find the values of \( x \) and \( y \).

**E. Trapezium**

**Activity 5.9**

1. Find four sticks of length 5 cm, 10 cm, 3 cm and 3 cm. Construct a quadrilateral by joining the sticks end to end and by making the 5 cm and 10 cm sticks at the opposite sides.

2. Copy the image of the shape on piece of paper. Measure all the interior angles. What do you investigate?

3. How do you name this type of quadrilateral?
Definition:

A trapezium (trapezoid) is a special type of quadrilateral in which exactly one pair of opposite sides is parallel.

- The parallel sides are called the bases of the trapezium.
- The distance between the two parallel bases is known as the height (or altitude) of the trapezium. The height is always perpendicular to the two bases.

The quadrilateral given is a trapezium where;

- \( \overline{AB} \) and \( \overline{DC} \) are bases of the trapezium,
- \( \overline{AE} \) is the height (altitude) of the trapezium,
- \( \overline{AD} \) and \( \overline{BC} \) are non parallel sides called legs of the trapezium.

Example 5.11

Construct a trapezium \( ABCD \) using ruler, protractor, pair of compasses and the given information below.

Given: \( AB \parallel CD \), \( AB = 8\text{cm} \), \( BC = 5\text{cm} \), \( m(\angle A) = 60^\circ \) and \( m(\angle B) = 85^\circ \).

Solution:

Step 1: Draw a line segment \( AB \).

Step 2: Using a protractor, construct \( m(\angle A) \) and \( m(\angle B) \) with the given degree measures as in the figure.

Step 3: Mark point \( C \) on the side of \( \angle B \) such that \( BC = 5\text{cm} \).
Step 4: Draw a line through C and parallel to AB so that it intersects the side of \( \angle A \) at point D.

Therefore, ABCD is the required trapezium.

**Definition:**

A trapezium is said to be **isosceles trapezium** if it has a pair of equal base angles. If a trapezium is isosceles, then

- The pair of non-parallel sides are equal.
- Diagonals are equal

**Example 5.12**

Find the measure of the unknown angles in each isosceles trapezoid.

**Solution:**

a. \( c = \theta(\angle ADC) = 55^\circ \)

Therefore, angle \( c = 55^\circ \)

And, \( c + b = 180^\circ \)

\[ 55^\circ + b = 180^\circ \]  

Substitution

\( b = 180^\circ - 55^\circ = 125^\circ \)

Similarly, \( a = 125^\circ \)
b. \( d + m(\angle LON) = 180^\circ \)  
\[ d + 135^\circ = 180^\circ \]
\[ d = 180^\circ - 135^\circ = 45^\circ \]

\[ e = d = 45^\circ \]  
Base angles of isosceles trapezium

\[ f + e = 180^\circ \]  
why?

\[ f + 45^\circ = 180^\circ \]

\[ f = 180^\circ - 45^\circ = 135^\circ \]

Example 5.13

The diagram to the right shows isosceles trapezoid ABCD with \( \overline{AB} \parallel \overline{DC} \) and \( AD=BC \). If \( AD = 4x \) and \( BC = 3x+5 \), what is the value of \( x \)? What is the length of \( BC \)?

Solution:

Given: \( AD = BC, AD = 4x \) and \( BC = 3x+5 \)

\[ 4x = 3x+5 \]

\[ 4x-3x = 5 \]

\[ x = 5 \text{ units} \]

Therefore, \( BC = 3x+5 = 3(5)+5 \)

\[ BC = 20 \text{ units} \]

\[ BC = 20 \text{ units} \]

F. Kite

Activity 5.10

1. Find two pairs of stick, one pair with length 5cm each, and the other pair with length 3cm each. Construct a quadrilateral by joining equal length sticks end to end.

   a. Copy the image of the shape on piece of paper. Name the quadrilateral formed \( ABCD \). Measure all the interior angles. What do you investigate?

   b. Draw the diagonals. Measure the angles formed when the diagonals bisect each other at point \( O \). Measure lengths of \( OA, OB, OC \) and \( OD \)? What do you investigate?

2. Do you remember what you used to play by folding a paper like airplane and throwing in the air? What shape is it?
UNIT 5: PERIMETER AND AREA OF PLANE FIGURES

Definition:

A *kite* is a quadrilateral with two adjacent sides congruent and the other two sides are also congruent.

Properties of Kite

Quadrilateral ABCD is a kite:

a. The two adjacent sides of a kite are equal in length (AB = BC and CD = AD)

b. $\overline{AC}$ and $\overline{BD}$ are diagonals of the kite. Diagonals of a kite are perpendicular to each other at point M.

c. $\overline{BD}$ is a bisector of the other diagonal $\overline{AC}$ but $\overline{AC}$ may not be a bisector of $\overline{BD}$.

d. One of the diagonals ($\overline{BD}$ in the figure) bisects the opposite angles. (that is $m(\angle CBD) = m(\angle ABD)$ and $m(\angle CDB) = m(\angle ADB)$)

Example 5.14

Find the value of the variable $x$ for the given kite.

Solution:

\[2x + 10x - 6 + 90^\circ = 180^\circ\] why?

\[12x = 96^\circ\]

\[x = \frac{96^\circ}{12} = 8^\circ\]

Example 5.15

For what value of $x$ is the figure below a kite?
Solution:
\[ 4x + 1 = 17 \text{ and } 6x - 3 = 21, \quad \text{why?} \]
\[ 4x = 17 - 1 \text{ and } 6x = 21 + 3 \]
\[ 4x = 16 \text{ and } 6x = 24 \]
\[ x = 4 \text{ and } x = 4 \]
Therefore, \( x = 4 \)

Exercise 5.6

1. Look at the figure to the right and answer the questions that follow.
   a. List all possible triangles in the figure
   b. List all possible quadrilaterals in the shaded figures.

2. Mulunesh says the quadrilateral ABCD in the figure to the right is a parallelogram if \( m(\angle CAD) = m(\angle ACB) \). Is she correct? Why?

3. Write all, some or no to complete the following sentences
   a. __________ squares are rectangles.
   b. __________ squares are rhombus
   c. __________ kites are rhombus
   d. __________ kite are square
   e. __________ parallelogram are rectangles

4. In each of the following, choose at least one word from parallelogram, rectangle, rhombus, trapezoid, kite, or square, so that the resulting sentence is true. If none of the words makes the sentence true, answer “none.”
   a. __________ a quadrilateral whose diagonals are equal and bisect each other.
   b. __________ a quadrilateral whose diagonals are perpendicular and bisects each other.
   c. __________ a quadrilateral whose diagonals are equal, perpendicular and they bisect each other.
   d. __________ a quadrilateral whose pair of opposite sides is parallel and equal.
5. In the figure to the right, angles are given as shown. Then
   a. Find the value of y.
   b. Find the sizes of the angles in the quadrilateral.

6. Is the information given in the diagram below enough to say quadrilateral PQRS is an isosceles trapezoid? Why?

7. Identify and correct the error in the following statement. \( \angle B \) and \( \angle C \) are supplementary angles, so, \( \overline{AB} \parallel \overline{CD} \). Therefore, ABCD is a parallelogram.

8. Construct a quadrilateral ABCD with \( \overline{AB} \parallel \overline{CD} \) and \( AB = 6\text{cm}, \ BC = 3\text{cm}, \ m(\angle A) = 50^\circ \) and \( m(\angle B) = 80^\circ \).

5.3 Perimeter and Area of Quadrilaterals

**Activity 5.11**

1. Measure the length of all the four sides of your mathematics textbook front cover page.
   a. Add up all your measures.
   b. What do you call this sum?
   c. Using your measurements, find the area of the cover page?
   d. Explain the difference between your results in b) and c).

2. Think about the playground or football field of your school.
   a. If you run all around it, what do you call the distance you covered?
   b. If you run all-round the rectangular track shown below three times, what is the total distance you covered?
In the above activity, you are measuring the perimeter of the book cover. The distance all around your school ground is also the perimeter of the playground.

The perimeter (p) is the distance or length all around a two dimensional shape or figure. Perimeter of a four sided figure is obtained by adding measures of all the four sides. The perimeter of any n-sided plane figure is obtained by simply adding the lengths of the n sides.

That is, for n-sided polygon, its perimeter is:

\[ P = s_1 + s_2 + s_3 + \cdots + s_n \], where, \( s_1 \), \( s_2 \), \( s_3 \), \ldots, \( s_n \) are lengths of the sides.

**Example 5.16**

Find the perimeter of the figure to the right.

**Solution:**

Since the figure is 7-sided, we have to add all the lengths of the seven sides.

Therefore, \( P = 8\text{m} + 17\text{m} + 6\text{m} + 8\text{m} + 6\text{m} + 10\text{m} + 12\text{m} = 67\text{m} \)

**Example 5.17**

A man wants to fence his rectangular garden which is 15m long and 8m wide. Find the minimum length of fencing material he needs to buy.

**Solution:**

The minimum length of fencing material he needs to buy is the perimeter of the garden. Perimeter = \( 15\text{ m} + 8\text{m} + 15\text{m} + 8\text{m} = 46\text{m} \)

**Exercise 5.7**

1. Find the perimeter of the figure below.

![Diagram of a figure with sides 22cm, 14cm, 9cm, 11cm, and 33cm.]

2. Ten equilateral triangles with side length 1 unit each are arranged as shown below. The perimeter of the whole figure is 12 units. What will be the perimeter of the figure, if it is extended to include a total of 30 such triangles?
Area of plane figures

Activity 5.12

1. The figure below is a rectangular region. Each division is 1 unit square.
   a. Count the small unit squares. How many are there?
   b. How many unit squares are there vertically and horizontally? Multiply these results and compare with the result you get in (a)?

2. Based on the above task, define the area of a plane figure.

3. If the area of your classroom is 42m², how do you explain this?

**Area** of a closed figure is the number of square units inside that closed figure. The term ‘area’ refers to the amount of space inside the boundaries of a figure. We frequently use the concept of area in different aspects of our daily life. For instance, if you want to paint the walls of your room, you have to know the area to be painted so as to buy the adequate amount of paint.

Calculating area of a room is important to determine the number of tiles needed or the size of a new carpet to be fitted into the room. Many people such as builders, architects, farmers or engineers need to calculate areas as part of their daily job.

If you ask an architect to design a new house for you, he/she must first know the area of the land where you want to build the house. Farmers working in their vegetable fields use area to decide the amount of seeds or fertilizers to use.

Look at this figure which has been placed over a grid. We can count the squares inside the rectangle and thus obtain an area of 15 square units.
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Check!
Count the number of squares in the rectangle. Are they all complete squares?

Check!
Is the region bounded by the triangle covered by complete squares?

However, we do not always have complete squares and thus it may be difficult in counting the number of squares. So we are using different formulas to calculate the area of different plane figures.

Perimeter and Area of a Rectangle

Area of a rectangle is the number of unit squares that filled the rectangle. This is also obtained by the product of its length and its width.

Area and perimeter of Rectangle

Area (A) of a rectangle = length × width
\[ A = \ell \times w \]
Perimeter (p) of a rectangle = 2 (length + width)
\[ p = 2(\ell + w) \]

Perimeter and Area of a Square

Since square is a rectangle and lengths of sides of a square are equal, area and perimeter of a square is obtained by taking the length and width of the rectangle which are equal.
Area and perimeter of Square

Area of a square; \( A = \text{length} \times \text{width} \)
\[ = \text{length} \times \text{length} \]
\[ = (\text{length})^2, \ell = w \]
\[ A = \ell^2 = s^2, \text{ } s = \text{side length}. \]
Perimeter of a square; \( p = 4\ell \) or \( p = 4s \),

\[ \text{Example 5.18} \]

Find the perimeter and area of a rectangle with length 9m and width 5m.

\[ \text{Solution:} \]

Given: \( \ell = 9 \text{m}, w = 5 \text{m}. \)

Required: Perimeter and Area of the rectangle.

\[ P = 2(\ell+w) = 2(9+5) \]
\[ = 2(14) = 28 \text{m}. \]
\[ A = \ell \times w = 9 \times 5 = 45 \text{m}^2. \]

\[ \text{Example 5.19} \]

Find the perimeter and area of a square with side 21cm

\[ \text{Solution:} \]

Given: \( s = 21 \text{cm}. \)

Required: Perimeter and Area of the square.

\[ P = 4s = 4 \times 21 = 84 \text{cm} \]
\[ A = s^2 = (21 \text{cm})^2 = 441 \text{cm}^2 \]

\[ \text{Example 5.20} \]

Calculate the perimeter and area of the figure to the right

\[ \text{Solution:} \]

\[ P = 6\text{cm} + 4\text{cm} + 2\text{cm} + 3\text{cm} + 4\text{cm} + 7\text{cm} \]
\[ P = 26 \text{cm} \]

To find the area, let us first divide the figure into two parts.
The rectangle A has an area of
\[4\text{cm} \times 7\text{cm} = 28\text{cm}^2\]
The rectangle B has an area of
\[2\text{cm} \times 4\text{cm} = 8\text{cm}^2\]
When we combine these we will have a total area of the figure.
Area of A + Area of B = 28 cm² + 8 cm²
\[= 36\text{ cm}^2\]

Example 5.21

If the length of a rectangle is twice its width, and the perimeter is 36cm, then find the length, width and area of the rectangle

Solution:

Given: \(\ell = 2w\), \(p = 36\text{cm}\)

Required: \(\ell\), \(w\) and \(A\)

\[P = 2(\ell + w)\]
\[36\text{ cm} = 2(\ell + w),\]
\[\ell + w = 18\text{ cm}\] Dividing both sides by 2
\[2w + w = 18\text{ cm}\] Substitution
\[3w = 18\text{ cm}\]
\[w = 6\text{ cm}\]

Hence, \(\ell = 2w = 2 \times 6\text{ cm} = 12\text{cm}\)

Therefore, \(\ell = 12\text{cm}\) and \(w = 6\text{cm}\)

\[A = \ell \times w = 12\text{cm} \times 6\text{cm}\]
\[A = 72\text{cm}^2\]

Note that, we can represent the length as the base and the width as height of the rectangle. Therefore, we can write the formula for the area of a rectangle as:

Area of a rectangle = base \(\times\) height

That is, \(A = bh\)
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Exercise 5.8

1. In both figures below parts of blocks have 1cm length of side. For each figure determine the area in square centimeters and the perimeter in centimeters?

   a) 

   b)

2. A farmer has 36 meters of fencing material for his rectangular shape farm land whose one side 22m is along a river. He plans to use 22 meters of the river for one side and to fence the remaining part of the farm land.
   a. Draw and label the diagram of the farm land.
   b. Find the width of the farm land
   c. Calculate the area of the farm land.

3. A wall of 11m by 10m has an opening of 3m by 3m. What would be the cost of painting of the wall at Birr 150 per square meter?

4. Playing cards are arranged to form a square of side 24cm. The same playing cards are then rearranged to form a rectangle of length 36cm.
   a. Find the width of the rectangle.
   b. Which of the two figures will have greater area? The square or the rectangle.
   c. Which of the two figures will have greater perimeter?

Problem Solving

In the figure to the right, the large square ABCD has been divided into 9 squares. The lengths of the sides of two squares are given. Find the perimeter and area of the large square ABCD.
Area of a triangle

**Activity 5.13**

1. Draw a rectangle of length 20cm and width 12cm on a hard paper.
2. Find the area of the rectangle.
3. Cut out the rectangle carefully.
4. Draw one diagonal of the rectangle, cut the rectangle into two triangles along the diagonal.
5. Check the sizes of the triangles by fitting one onto the other.
6. What do you know about the area of one of the triangles?
7. Try to write the area formula for the triangle.

The area of a triangle tells us how many unit squares the triangle contains. To find the area of a triangle, you need to know the base and the height of the triangle.

To compute the area of a right triangle, you will apply the knowledge of the area of rectangles (as you did in the above activity). If the sides of the rectangle are ‘b’ and ‘h’ units, then the area $A$ of the rectangle is given by

$$ A = bh. $$

We also know that each diagonal divides the rectangle into two equal triangles. Hence, the area $(A)$ of the right triangle ABC is given by

$$ A = \frac{1}{2}bh $$

**Theorem**

The area $A$ of any triangle whose base is $b$ and the altitude to this base is $h$ is given by

$$ A = \frac{1}{2}bh $$
Example 5.22

Find the area of the triangle given in the figure.

**Solution:**

Given: \(b = 14\text{cm}\) and \(h = 12.1\text{cm}\)

\[
A = \frac{1}{2}bh = \frac{1}{2} \times 14\text{cm} \times 12.1\text{cm} = 84.7\text{cm}^2
\]

Example 5.23

The area of a triangular shaped window of a building is \(25\text{m}^2\). If base of the window is \(5\text{m}\), then what is the height of the window?

**Solution:**

Area of the window = \(25\), \(b = 5\text{m}\)

\[
A = \frac{1}{2}bh = \frac{1}{2}(5\text{m})h = 25\text{m}^2
\]

\[(5\text{m})h = 50\text{m}^2\]

\[h = 10\text{m}\] \hspace{1cm} \text{Dividing both sides by 5m}

Therefore, the height of the window is \(10\text{m}\).

Example 5.24

Find the area of the shaded part of the rectangle.

**Solution:**

Area of shaded part = area of rectangle – area of triangle

\[
A(\text{shaded}) = bh - \frac{1}{2}bh = (12\text{m})(6\text{m}) - \frac{1}{2}(12\text{m})(6\text{m})
\]

(Base and height of the rectangle and triangle are the same)

\[= 72\text{m}^2 - 36\text{m}^2 = 36\text{m}^2\]

Therefore, the area of the shaded part of the rectangle is
**Example 5.25**

In the figure to the right, $\overline{CD} \perp \overline{AB}$ with $AB = 12\text{cm}$ and if $E$ is on $\overline{CD}$ and $CE = 3\text{cm}$, then what is the area of the shaded region?

**Solution:**

Let $DE = x \text{ cm}$, then $DC = (x + 3) \text{ cm}$.

Area of the shaded region $= \text{Area of } \triangle ABC - \text{Area of } \triangle ABE$.

$$= \frac{1}{2} \times 12\text{cm} \times (x + 3) \text{ cm} - \frac{1}{2} \times 12\text{cm} \times x\text{cm}$$

$$= 6(x + 3) \text{ cm}^2 - 6x\text{cm}^2$$

$$= (6x + 18) \text{ cm}^2 - 6x\text{cm}^2$$

$$= 6x\text{cm}^2 - 6x\text{cm}^2 + 18\text{cm}^2$$

$$= 18\text{cm}^2$$

Therefore, the area of the shaded region is $18\text{cm}^2$.

**Exercise 5.9**

1. In the figure given to the right, if $ABCD$ is a rectangle, then what is the area of the shaded part?

2. Triangle $ABC$ is shown below in two positions, with two different bases and altitudes. One of these altitudes falls outside the triangle. Determine the area of each triangle.
3. ABCD is a rectangle with AB = 10cm and AD = 6cm. What is the area of the shaded part?

Area of a parallelogram

**Activity 5.14**

1. On a grid (squared) paper, draw a parallelogram by choosing any length you like and cut out the parallelogram. Is the region covered by the parallelogram filled with complete squares? How can you find the area of such a region?

2. Draw the height of the parallelogram from a vertex and cut the parallelogram along the height.
   - Take the left cut part and place to the right side of the parallelogram so that the non-horizontal sides fit together.
   - Observe the new formed figure. What do you say about the figure? What type of quadrilateral is it?

3. Is there a change in size between the first parallelogram and the new one?

4. Can you find the area of the new figure? How do you compare the areas of the two figures?

From the activity above, you can understand that the area of the given parallelogram is equivalent to the area of the figure you formed (a rectangle), \((\text{base})(\text{height})\) that is. Similarly, you can derive the same formula as follows.

You know that the diagonal divides the parallelogram into two equal triangles.
So, \( a(ABCD) = a(\triangle ADC) + a(\triangle ABC) \)
\[
= \frac{1}{2} (DC) (AE) + \frac{1}{2} (AB) (BF)
\]
\[
= \frac{1}{2}bh + \frac{1}{2}bh, \quad DC = AB = b \quad \text{Why?}
\]
\[= bh\]

Therefore, area of a parallelogram is given by:

\[ A = \text{(base)} \times \text{(height)} = bh \]

**Example 5.26**

The area of a parallelogram is 98 cm\(^2\). Find its altitude if the base is 14cm.

**Solution:**

\[ A(\text{parallelogram}) = bh \]
\[ 98\text{cm}^2 = 14\text{cm} \times h \quad \text{Substitution} \]

Then, \[ h = \frac{98\text{cm}^2}{14\text{cm}} = 7\text{cm} \]

Therefore, the height of the parallelogram is 7cm.

**Example 5.27**

Suppose the base and height of a parallelogram are both doubled. How does the area of the parallelogram change?

**Solution:**

Let the base and height of the parallelogram be \( b \) and \( h \) respectively.

\[ A(\text{parallelogram}) = bh, \text{ but } b \text{ and } h \text{ are doubled, so} \]
\[ A(\text{parallelogram}) = (2b)(2h) = 4bh \]

Therefore, the area of the parallelogram is 4 times the original one.

**Example 5.28**

Ato Jemal is a hard working farmer. He has a parallelogram shaped irrigated land with area of 1800m\(^2\). The lengths of the two adjacent sides of the land are 50m and 40m. Find the shortest distance between the opposite sides of the land.
Solution:

Given the shape of the land is a parallelogram.

The distance between the two opposite sides is the height of the figure and let it be $h_1$ and $h_2$. Since $b_1 = 50\text{m}$ and $b_2 = 40\text{m}$

$$A(\text{land}) = b_1 \cdot h_1 = b_2 \cdot h_2$$

$$1800\text{m}^2 = b_1 \cdot h_1 = b_2 \cdot h_2 \quad \text{(have same area)}$$

$$1800\text{m}^2 = 50h_1 = 40h_2$$

$$1800\text{m}^2 = 50h_1 \text{ and } 1800\text{m}^2 = 40h_2$$

$$h_1 = \frac{1800\text{m}^2}{50\text{m}} \quad \text{and} \quad h_2 = \frac{1800\text{m}^2}{40\text{m}}$$

$$h_1 = 36\text{m} \quad \text{and} \quad h_2 = 45\text{m}$$

Thus, the distances between the two pairs of opposite sides are 36m and 45m. The shortest distance is 36m.

Exercise 5.10

1. In the figure to the right, length AP and length AQ are altitudes of the parallelogram ABCD.
   
a. If AQ = 4cm, CD = 5cm, find the area of ABCD.

b. If the area of ABCD = 24, AB=6cm then find AQ.

c. If AB = 5cm, AP = 4cm, AD = 6cm, then find AQ.

2. Describe the relationship between the area of a parallelogram and the area of a triangle with the same height and base. Explain.

3. An architect has drawn the ground floor of a building, as shown to the right. Find the area of the floor.
4. In the figure to the right, PQRS is a parallelogram with length of $PQ = 28\text{ cm}$ and the distance between $PQ$ and $SR$ is $16\text{ cm}$. Find:
   a. The area of PQRS
   b. The area of $\Delta SQR$ and $\Delta PQS$
   c. What did you observe? Are the areas of the two triangles equal?

**Area of a Rhombus**

**Activity 5.15**

Rhombus is a parallelogram with length of all sides equal; diagonals are perpendicular bisectors of each other.

1. Suppose a rhombus with its diagonals is given as shown. How can you find its area?
2. If $d_1 = 12$ units and $d_2 = 10$ units, then what will be the area of the rhombus?

From the activity, you can understand that the area of the rhombus is equivalent to the sum of the areas of the four right-angled triangles formed by the diagonals. The area of one of the triangles is formulated as: $\frac{1}{8} d_1 d_2$.

**Note:**

Area of rhombus with diagonals $d_1$ and $d_2$ is given by: $A = \frac{1}{2} d_1 d_2$

**Example 5.29**

If the diagonals of a rhombus are $8\text{ cm}$ and $6\text{ cm}$, then find the perimeter and area of the rhombus.

**Solution:**

Area of a rhombus $= \frac{1}{2} d_1 d_2$

$= \frac{1}{2} \times 8\text{ cm} \times 6\text{ cm} = 24\text{ cm}^2$
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To find the perimeter of the rhombus, consider the right angle triangle $\Delta AOB$.

$$(AO)^2 + (OB)^2 = (AB)^2$$

$$(AB)^2 = (3\text{cm})^2 + (4\text{cm})^2$$

$$(AB)^2 = 9\text{cm}^2 + 16\text{cm}^2 = 25\text{cm}^2$$

$AB = 5\text{cm}$

Therefore, perimeter of the rhombus is $P = 4 \times 5\text{cm} = 20\text{cm}$

**Exercise 5.11**

1. Find the perimeter of a rhombus with diagonals 5m and 12m.
2. Find the area of the rhombus having each side 17cm and one of its diagonals is 16cm.
3. The diagonals of a rhombus are 16cm and 30cm. Find the length of each side and its perimeter?
4. The perimeter of a rhombus is 40cm. The length of one of the diagonals is 12cm. How do you find the area of the rhombus?
5. The floor of a building consists of 2000 tiles which are rhombus shaped and each of its diagonals are 40cm and 25cm in length. Find the total cost of polishing the floor, if the cost per m² is 35BIRR.

**Area of a Trapezium**

From our previous discussion you recall that a trapezium is a quadrilateral in which a pair of opposite sides are parallel. Now, we will see how to find the area of a trapezium.

**Activity 5.16**

On a square paper draw a trapezium $ABCD$ with parallel bases $b_1$ and $b_2$, and its height $h$ (see the figure to the right).

a. Divide the trapezium into two triangles, namely $\Delta ABC$ and $\Delta ACD$.

b. Write the area formula for $\Delta ABC$ and $\Delta ACD$

c. Add the areas of the two triangles. What do you observe?

d. Can you write the area formula for the trapezium you draw?
**Area formula for a Trapezium**

If the lengths of the bases of a trapezium are denoted by \(b_1\) and \(b_2\) and its altitude is denoted by \(h\), then the area \(A\) of the trapezium is given by:

\[
A = \frac{1}{2} (b_1 + b_2) \cdot h
\]

**Example 5.30**

What is the area of the trapezium shown in the figure?

**Solution:**

Let \(b_1 = 15\, \text{cm}, \ b_2 = 9\, \text{cm}\) and \(h = 6\, \text{cm}\)

\[
A = \frac{1}{2} (b_1 + b_2) \cdot h = \frac{1}{2} (15\, \text{cm} + 9\, \text{cm}) \cdot 6\, \text{cm}
\]

\[= 24\, \text{cm} \times 3\, \text{cm} = 72\, \text{cm}^2\]

**Example 5.31**

A parking lot in Bahir Dar has trapezoidal shape with area of 30\(\, \text{m}^2\). If the distance between the two parallel sides is 5\(\, \text{m}\) and one of the bases is 8\(\, \text{m}\), find the length of the other parallel side.

**Solution:**

\[
A = \frac{1}{2} (b_1 + b_2) \cdot h , \text{ where } A = 30\, \text{m}^2, \text{ and } h = 5\, \text{m}
\]

\[
30\, \text{m}^2 = \frac{1}{2} (b_1 + 8\, \text{m}) \cdot 5\, \text{m}
\]

\[60\, \text{m}^2 = 5b_1, \text{m} + 40\, \text{m}^2
\]

\[
\frac{20\, \text{m}^2}{5\, \text{m}} = \frac{5b_1, \text{m}}{5\, \text{m}}
\]

\[4\, \text{m} = b_1\]

Therefore, the length of one of the parallel sides is 4\(\, \text{m}\).
Area of a kite

Remember that a kite has two pairs of equal adjacent sides and its diagonals meet at right angle.

Activity 5.17

In doing the activity on trapezium, remember that to find area formula of the trapezium, you divided the trapezium into two triangles.

a. For the area of a kite, what methods do you think will help you to derive the area?

b. Discuss in a small group and try to design and write the area formula for a kite?

Example 5.32

Find the area of the given figure.

Solution:

Since two pairs of adjacent sides are equal, the figure is a kite.

Given: \( d_1 = 8\text{cm} \) and \( d_2 = 5\text{cm} \).

The area of a kite is given by \( A = \frac{1}{2} \times d_1 \times d_2 \).

Then, \( A = \frac{1}{2} \times 8\text{cm} \times 5\text{cm} = 20\text{cm}^2 \)

Exercise 5.12

1. The area of a trapezium is 35cm\(^2\). Find its altitude if the bases are 6cm and 8cm.

2. The angles of a quadrilateral are in the ratio of 1: 2: 3: 4. What is the measure of each angle of the quadrilateral?

3. If the area of the trapezium ABCD is 30cm\(^2\), then find the value of \( b_1 \).
4. Calculate the area of the figure below

5. Draw a trapezoid with parallel bases and, and height h. Then fold the trapezium to be base onto base. Cut the trapezoid along this fold line and form a parallelogram as shown below.

a. Find the area of the newly formed parallelogram (right side of the figure).

b. Is there a relationship between the area of the trapezoid and the area of the new formed parallelogram?

c. Based on your results in (a) and (b), find area of the trapezoid.

d. Find any other way of deriving the area formula of a trapezium?

5.4 Perimeter and Area of a Circle

In this sub section we will discuss about the circumference (perimeter) and area of a circle.

Activity 5.18

1. Give examples of circular objects that you have observed.
2. Do you know traditional way of drawing a circle?
   Describe some.
3. What materials are used to draw circles in a mathematical way?
4. Draw a circle and indicate different terms related to a circle.
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Look at the circle on the paper below. We can fold it twice to get 2 fold lines as shown below.

The point where the 2 fold lines meet is called the Centre of the circle.

Note:

1. A circle is the set of all points in a plane that are the same distance from a given point called the center. A circle is usually named by its center.
   
   The circle to the right can be named as “circle O”.

2. The diameter (d) of a circle is a line segment through the circle and joining any two points of the circle.

   The radius (r) of a circle is the distance from the center to any point on the circle.

   In circle O, \( OA, OB \) and \( OF \) are radii (plural form of radius).

3. A chord of a circle is any line segment that joins two points of the circle. \( CD \) and \( AB \) are chords of circle O. A diameter of a circle is the longest chord of that circle.

5.4.1 Perimeter (Circumference, C) of a Circle

As we have seen before, the distance all-round the plane figure is perimeter. For a circle, this perimeter is named as circumference of the circle. The circumference (C) of a circle is therefore, the distance around the circle.

Activity 5.19

1. Use ruler, string, circular objects of various sizes for your work.
   
   a. Bring a circular object.
   
   b. Use a ruler to measure the diameter of your circular object. Record your finding.
   
   c. Wrap a string around the circular object once. Mark the string where it meets itself.
   
   d. Lay the string out straight, measure and then record the length of the string with your ruler. Can you call this the circumference of the circle?
e. Divide the measure of the circumference obtained in (d) by the measure of the diameter obtained in (b). Record your answer.

f. Repeat this activity with circular objects of various sizes.

2. Compare the results you obtained in (e) and (f). What do you observe or find?

3. How is the circumference related to the diameter?

4. What is your conclusion?

From the activity, you can understand that the ratio of the circumference to the diameter of each of the circles you considered approaches a number very closer to 3.14 called \( \pi \).

Therefore, the circumference divided by the diameter of any circle is given as: \( \frac{c}{d} = \pi \), (where \( \pi \) is Greek letter (read as pi)). Most of the time the rational numbers we used for approximately \( \pi \) are 3.14 or \( \frac{22}{7} \).

**Circumference of a Circle**

For any circle with diameter \( d \), or radius \( r \), its circumference is given by:

\[
C = \pi d \quad \text{or} \quad C = 2\pi r
\]

**Example 5.33**

Find the circumference of a circle with diameter 17m.

**Solution:**

\[
C = \pi d, \quad \text{and} \quad d = 17m.
\]

Then, \( C = \pi d = \pi \times 17m = 3.14 \times 17 \approx 53.38 \text{m} \),

Therefore, the circumference of the circle is 17\( \pi \) m or approximately 53.38m.
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Example 5.34

The basement of the school flag pole is a circular concrete. The circumference of the basement is 22m. Find the radius of the basement. (Use \( \frac{22}{7} \) for \( \pi \))

**Solution:**

Given: \( C = 22\text{m} \)

\[
C = 2 \pi r
\]

\[
22\text{m} = \left(2 \times \frac{22}{7} \times r\right) = \frac{44}{7} r
\]

\[
22\text{m} \times \frac{7}{44} = \frac{44}{7} r \times \frac{7}{44}
\]

\[3.5 \text{m} = r\]

Therefore, the radius of the basement is 3.5m.

Example 5.35

The figure below is formed by a semicircle and a straight line.

What is its perimeter? (Use \( \pi = \frac{22}{7} \))

**Solution:**

Diameter of circle, \( d = 7\text{m} \)

\[
\frac{1}{2} \text{ of the circumference of a circle} = \frac{1}{2} \times \pi \times d
\]

\[= \frac{1}{2} \times \frac{22}{7} \times 7\text{m} = 11\text{m}.
\]

Perimeter of the semicircle \( = \frac{1}{2} \) of the circumference of a circle + length of the diameter.

\[= 11\text{m} + 7\text{m} = 18\text{m}\]

5.4.2 Area of a Circle

Do you remember what we said about the area of a plane figure? It is the number of square units that cover the plane figure. We can use a square grid to find the approximate area of a circle.
Let area of each small square be 1 cm². Number of grid squares covered by the quadrant is 39.

Area of quadrant = 39 × 1 cm² = 39 cm²
Area of circle = 39 cm² × 4 = 156 cm²

Let us look at a more accurate way of finding the area of a circle.

**Activity 5.20**

Materials needed: paper, compass, ruler or straight edge, scissors, pencil.

1. Draw a circle and several radii that separate the circle into **equal-sized** sections. (Let $r$ units represent the length of the radius and $C$ units represent the circumference of the circle).
2. Cut out each section of the circle.
3. Reassemble the sections in the form of a parallelogram.

![Diagram of a circle divided into sections and reassembled into a parallelogram]

4. What is the base length of this “parallelogram”? How about length of its height?
5. Find the area of the parallelogram. Remember area of a parallelogram is: $A = bh$.
6. How could you use this formula to find the area of a circle?

The base of the parallelogram shown on the activity above is half the circumference of the circle

$$\frac{1}{2} C = \frac{1}{2} 2\pi r = \pi r$$

The height of the parallelogram is the length of the radius. Substitute this information into the formula for the area of a parallelogram.

$$\text{Area of a circle} = \text{area of the parallelogram} = \frac{1}{2} C \times r = \pi r \times r = \pi r^2$$
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Area of a Circle

For any circle with radius $r$ units, the area of the circle is given by: $A = \pi r^2$

Example 5.36

Find the area of a circle of diameter 10m. (Take $\pi = 3.14$)

Solution:

$d = 10m$, so that $r = 5m$.

$A = \pi r^2 = 3.14 \times (5m)^2$

$= 3.14 \times 25m^2 = 78.5 \text{ m}^2$.

Example 5.37

Find the length of the radius of a circular watch if its area is 38.5cm$^2$

Solution:

Given: $A = 38.5 \text{ cm}^2$, required: $r = ?$

$A = \pi r^2$

$38.5 \text{ cm}^2 = 3.14 r^2$

$\frac{38.5 \text{ cm}^2}{3.14} = r^2$

$12.25 \text{ cm}^2 = r^2$

$r = \sqrt{12.25 \text{ cm}^2} = 3.5\text{ cm}$

Calculator Hint

$\pi$ is an important number in mathematics that has its own key on a calculator. What is displayed on your calculator when you press $\pi$?

Example 5.38

In figure given Find the area of the shaded region

Solution:

Area of the shaded region $= \text{area of circle} - \text{area of square}$

Area of the circle: $A_c = \frac{\pi d^2}{4}, r = \frac{d}{2}$

$= \frac{(3.14)(35\text{ cm})^2}{4} = \frac{(3.14)1225\text{ cm}^2}{4} = \frac{3846.5}{4} \text{ cm}^2$

$= 961.625\text{ cm}^2$
Area of the square: \( A_s = s^2 = (8\text{cm})^2 = 64\text{cm}^2 \)
Area of shaded region \( A = A_c - A_s = 961.625 \text{ cm}^2 - 64 \text{ cm}^2 = 897.625 \text{ cm}^2 \)

**Exercise 5.13**

1. Complete the following table. (Take \( \pi = \frac{22}{7} \))

<table>
<thead>
<tr>
<th>Radius</th>
<th>Diameter</th>
<th>circumference</th>
<th>area</th>
</tr>
</thead>
<tbody>
<tr>
<td>14mm</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>14cm</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>176 cm</td>
<td></td>
<td>5,544</td>
</tr>
</tbody>
</table>

2. Tell how the circumferences of two circles relate if the diameter of one is twice as long as the diameter of the other.

3. The figure to the right is made up of a quadrant and a triangle. Find its perimeter. (Take \( \pi = 3.14 \))

4. Find the area of a semicircle whose radius is 2.4 cm.

5. If the radius of a circle is doubled, what happens to its area?

6. There are two gardens as shown on the figure to the right. One is square and the other is circular. Which garden would need the most fencing material to go around it? Which garden would have the greater area?

7. A road is constructed through hills making two half circles whose diameters are given in the figure. Find the length of the road from A to C. (Take \( \pi = 3.14 \)).
8. Eyob is a tailor. From a rectangular piece of cloth which is shown on the figure to the right the cut out another piece which has a triangular piece and a shape of a quadrant of a circle. Find
   a. The perimeter
   b. The area of the remaining piece of cloth. (Take $\pi = \frac{22}{7}$)

9. The floor of a swimming pool is formed by two quadrants and a rectangle. Find
   a. the floor area of the swimming pool.
   b. the cost of tiling if the tiling costs Birr 40 per square meter (Take $\pi = 3.14$)

10. Find the area of the figure below
**UNIT 5: PERIMETER AND AREA OF PLANE FIGURES**

**Unit Summary**

- Triangle is the simplest type of polygon.
- Triangles are classified as:

<table>
<thead>
<tr>
<th>Criteria</th>
<th>Types of triangles</th>
<th>conditions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sides</td>
<td>Equilateral triangles</td>
<td>all three sides equal</td>
</tr>
<tr>
<td></td>
<td>isosceles triangles</td>
<td>two sides equal</td>
</tr>
<tr>
<td></td>
<td>Scalene triangles</td>
<td>all three sides different in length</td>
</tr>
<tr>
<td>Angles</td>
<td>Acute angle triangle</td>
<td>all the three angles acute</td>
</tr>
<tr>
<td></td>
<td>Right angled triangle</td>
<td>one of the angles is right angle</td>
</tr>
<tr>
<td></td>
<td>Obtuse angled triangle</td>
<td>one of its angles is obtuse</td>
</tr>
</tbody>
</table>

- A quadrilateral is a four-sided geometric figure bounded by line segments
- A quadrilateral with equal diagonals may not be a rectangle
- A diagonal of a quadrilateral is a line segment that connects two opposite vertices
- A polygon is a simple closed path in a plane which is entirely made up of a line segment joined end to end.
- A convex polygon is a simple polygon in which each of the interior angles measure less than $180^\circ$
- A concave polygon is a simple polygon which has at least one interior angle of measure greater than $180^\circ$
- A diagonal of a convex polygon is a line segment whose end points are non – consecutive vertices of the polygon.
- The sum of the measures of all the interior angles of a polygon of $n$ - sided is given by $(n – 2)\times180^\circ$
- A chord of a circle is a line segment joining two points of a circle.
- A diameter of a circle is any chord that passes through the center of the circle.
### UNIT 5: PERIMETER AND AREA OF PLANE FIGURES

#### Quadrilateral Tree

![Quadrilateral tree diagram](image)

<table>
<thead>
<tr>
<th>Name</th>
<th>Shape</th>
<th>Area</th>
<th>Perimeter</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Rectangle</strong></td>
<td><img src="image" alt="Rectangle" /></td>
<td>$A = ab$</td>
<td>$P = 2(a + b)$</td>
</tr>
<tr>
<td><strong>Square</strong></td>
<td><img src="image" alt="Square" /></td>
<td>$A = s^2$</td>
<td>$P = 4s$</td>
</tr>
<tr>
<td><strong>Triangle</strong></td>
<td><img src="image" alt="Triangle" /></td>
<td>$A = \frac{1}{2}bh$</td>
<td>$P = a + b + c$</td>
</tr>
<tr>
<td><strong>Parallelogram</strong></td>
<td><img src="image" alt="Parallelogram" /></td>
<td>$A = bh$</td>
<td>$P = 2(a + b)$</td>
</tr>
</tbody>
</table>
UNIT 5: PERIMETER AND AREA OF PLANE FIGURES

<table>
<thead>
<tr>
<th>Trapezium</th>
<th>A = ( \frac{1}{2} (b_1 + b_2)h )</th>
<th>P = a + b_1 + c + b_2</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>![Trapezium Diagram]</td>
<td></td>
</tr>
<tr>
<td>Kite</td>
<td>A = ( \frac{1}{2}d_1d_2 )</td>
<td>P = a + b + c + d</td>
</tr>
<tr>
<td></td>
<td>![Kite Diagram]</td>
<td></td>
</tr>
<tr>
<td>Circle</td>
<td>A = ( \pi r^2 = \pi \frac{d^2}{4} )</td>
<td>C = 2\pi r</td>
</tr>
<tr>
<td></td>
<td>![Circle Diagram]</td>
<td></td>
</tr>
</tbody>
</table>

**Review Exercises**

1. Copy the chart and put a “√” mark in the box if the quadrilateral **always** has the given property. The first is done for you as example.

<table>
<thead>
<tr>
<th>No.</th>
<th>Property</th>
<th>Parallelogram</th>
<th>Rectangle</th>
<th>Rhombus</th>
<th>Square</th>
<th>Kite</th>
<th>Trapezoid</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>All sides are equal</td>
<td></td>
<td>√</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>B</td>
<td>Both pairs of opposite sides are equal</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>C</td>
<td>Both pairs of opposite sides are parallel</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>D</td>
<td>Exactly 1 pair of opposite sides are parallel</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>E</td>
<td>All angles are equal</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>F</td>
<td>Exactly 1 pair of opposite angles are equal</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>G</td>
<td>Diagonals are perpendicular</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>H</td>
<td>Diagonals are equal</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>I</td>
<td>Diagonals bisect each other</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
2. If the sum of the three angles of a quadrilateral is equal to $1\frac{2}{3}$ times the sum of the three angles of a triangle, what is the measure of the fourth angle of the quadrilateral?

3. The angles of a triangle are in the ratio of 2:3:5. What is the size of the smallest angle in degrees?

4. Look at the figure to the right and determine the value of $x$ and the measure of each angle.

5. A rectangular paper was cut as shown in the figure. What is the area the shaded part?

6. If triangle ABC and parallelogram ABED are formed between two parallel lines where points A and B are on line-k, and C, D and E are on line-$\ell$, then show that the area of the triangle is equal to half of the area of the parallelogram.

7. The opposite angles of a parallelogram are $(3x + 5)\degree$ and $(61 - x)\degree$. Find the measure of the four angles.

8. The area of the trapezoid to the right is $702\text{m}^2$ find the height of the trapezoid.
9. A baseball home plate can be divided into two trapezoids with the dimensions shown in the drawing. Find area of home plate.

10. On a floor, a pattern is made with tiles of trapezoid and square shapes as in the figure shown to the right. A side of the inner square measures 10 cm, and a side of the outer square measures 30 cm. What is the area of one of the trapezoid tiles?

11. Find areas and perimeters of the figures below.

12. When a wheel turns one complete round, we say that it makes one revolution. If the diameter of the wheel is 14 cm, find the distance the wheel moves in 2 revolutions. (Take $\pi = \frac{22}{7}$)

13. The figure shown to the right is made up of a semicircle and a triangle. Find area of the figure. (Take $\pi = 3.14$).
14. Iron rods a, b, c, d, e, and f are making a design in a bridge as shown below. If \( a \parallel b \), \( c \parallel d \), \( e \parallel f \), find the marked angles between

i. \( b \) and \( c \) 
ii. \( d \) and \( e \) 
iii. \( d \) and \( f \) 
iv. \( c \) and \( f \)

15. A sport field is prepared in the shape similar to the following figure. Where ABCD is a rectangle with \( AB = 100 \) m and \( AD = 40 \) m, \( \overline{AED} \) and \( \overline{BFC} \) are semicircles. Find

a. The area of the field
b. The perimeter of the field
UNIT - 6
CONGRUENCY OF PLANE FIGURES

Learning Outcomes:

After completing this unit, you will be able to:

- Understand conditions for congruency of two plane figures
- Identify congruent triangles by rules
- Use congruence concepts to solve real life problems
- Identify similar triangles

Key Terms

- Correspondence
- Congruency
- SSS
- SAS
- ASA
- RHS
- Similarity
UNIT 6: CONGRUENCY OF PLANE FIGURES

In the previous unit you have seen angles, perimeters and areas of triangles, quadrilaterals and polygons. In this unit, we will study congruence of plane figures specifically congruence of triangles and their real-life applications.

6.1 Congruency of Plane Figures

Activity 6.1

Look at the figures below; which pairs do you think have the same size and shape? Why?

A  B  C  D  E  F  G  H

a)  b)  c)  d)  e)  f)

Like carbon copies, photocopy machines are able to make reproductions of figures that have the same size and shape as the originals.

Given two plane figures, if one can be moved onto the other so that they coincide or if the two figures have the same size and shape, then we say they are congruent to each other. Art designers use templates to print the same shape figures again and again on papers or clothes. Such figures having the same shape and size are congruent figures. The word congruence is used with segments, angles, triangles and with other plane figures.

Note:

1. Two-line segments are congruent, if they have the same length,
2. Two angles are congruent, if they have the same angle measure,
3. The symbol “≈” stands for “is congruent to”.

Example 6.1

In the figures shown to the right, \( \triangle ABC \) is an equilateral triangle and \( \square DGFE \) is a square, in either of the two figures identify

a. Congruent sides
b. Congruent angles
Solution:

a. $\overline{AB}$, $\overline{BC}$ and $\overline{CA}$ are congruent (sides of an equilateral triangle)

$\overline{DG}$, $\overline{GF}$, $\overline{FE}$ and $\overline{ED}$ are congruent (sides of a square)

b. $\angle ABC$, $\angle BCA$ and $\angle CAB$ are congruent (interior angles of an equilateral triangle)

$\angle DGF$, $\angle GFE$, $\angle FED$ and $\angle EDG$ are congruent (interior angles of a square)

Example 6.2

The following figure looks like the design of part of the Bridge on Abay River. In the design of the bridge you can see line segments, angles and triangles, The points A(-6,0), B(0,3), C(6,0), D(-4,0), E(0,2), F(4,0), G(-2,0), H(0,1), P(2,0) and the origin O(0,0) are ends of line segments. Identify and list congruent pairs of line segments, angles and triangles.

![Diagram of the Bridge design](image_url)

Solution:

a. $\overline{AB}$ and $\overline{CB}$, $\overline{DE}$ and $\overline{EF}$, $\overline{GH}$ and $\overline{PH}$ are congruent line segments. Can you add more pairs of congruent line segments?

b. $\angle BAO$ and $\angle BCO$; $\angle EDO$ and $\angle EFO$ are congruent angles. Can you add more pairs of congruent angles?

c. $\triangle ABO$ and $\triangle CBO$ is a pair of congruent triangles. Can you add more pairs of congruent triangles?

d. There are also congruent quadrilaterals. Example ADEB is congruent to CFEB. Can you add more congruent quadrilaterals?
Exercise 6.1

1. From the given figures below, identify pairs of congruent figures, and explain why you think they are congruent.

6.2 Congruency of Triangles

Based on the above discussions on congruent line segments and congruent angles, can you explain what congruent triangles are?

Recall that if two line-segments have the same length, we say that they are congruent line segments. Similarly, if two angles have the same measure, we call them congruent angles. We indicate congruent segments and angles using the same marks on congruent parts.

Example 6.3

In the figure to the right congruency of pairs are indicated with marks as shown, therefore

a. \( \overline{AB} \cong \overline{CD} \) read as “line segment \( AB \) is congruent to line segment \( CD \)”

b. \( \angle PQR \cong \angle STU \) read as “angle \( PQR \) is congruent to angle \( STU \)”

Activity 6.2

Step 1: On a piece of grid paper, draw two triangles like the one below. Label the vertices as shown.
Step 2: Cut out the triangles. Put one triangle over the other so that the parts with the same measure match up.

Then

a. Identify all the parts of sides and angles that match or correspond.

b. Are these two triangles congruent? Why?

c. If they are congruent, what can you say about the corresponding sides and angles of the two triangles?

Note:

Look at the two triangles below, if $\triangle ABC$ and $\triangle DEF$ are congruent triangles, then:

a. $\overline{AB}$ and $\overline{DE}$

$b. AC$ and $\overline{DF}$

$c. BC$ and $\overline{EF}$ are corresponding sides

b. Points A and D, B and E, C and F are corresponding vertices

c. $\angle ABC$ and $\angle DEF$

$\angle ACB$ and $\angle DFE$

$\angle BAC$ and $\angle EDF$ are corresponding angles of the triangles

Congruent polygons have **congruent corresponding sides** and **congruent corresponding angles**. When you name congruent polygons, you must list corresponding vertices in the same order.
Example 6.4

In the figure below, the diagonal $\overline{OE}$ divide the parallelogram OREM into two congruent triangles, $\triangle MEO$ and $\triangle ROE$. Indicate the pairs of corresponding sides and corresponding angles.

**Solution:**

Given that $\triangle MEO \cong \triangle ROE$

a. $\overline{OM}$ and $\overline{ER}$
   $\overline{ME}$ and $\overline{RO}$
   $\overline{OE}$ and $\overline{EO}$ are corresponding sides

b. $\angle OME$ and $\angle ERO$
   $\angle MEO$ and $\angle ROE$
   $\angle EOM$ and $\angle OER$ are corresponding angles

**Definition:**

Two triangles ($\triangle ABC$ and $\triangle DEF$) are said to be congruent, if and only if, all pairs of corresponding sides and all pairs of corresponding angles are congruent. That is,

$\triangle ABC \cong \triangle DEF$ if and only if

\[
\begin{align*}
\overline{AB} & \cong \overline{DE} \\
\overline{AC} & \cong \overline{DF} \\
\overline{BC} & \cong \overline{EF}
\end{align*}
\]

$\angle ABC \cong \angle DEF$  Corresponding congruent sides

$\angle ACB \cong \angle DFE$  Corresponding congruent angles

$\angle BAC \cong \angle EDF$

Example 6.5

Show that the diagonal of a rectangle divides the rectangle into two congruent triangles.
Solution:

ABCD is a rectangle with diagonal \( \overline{AC} \)

i. \( AB \cong CD \) Opposite sides of rectangle

ii. \( BC \cong DA \) Opposite sides of rectangle

iii. \( AC \cong CA \) Common side of the two triangles

iv. \( \angle ABC \cong \angle CDA \) Right angles

v. \( \angle BAC \cong \angle DCA \) Alternate interior angles

vi. \( \angle ACB \cong \angle CAD \) Alternate interior angles

vii. Therefore, \( \triangle ABC \cong \triangle CDA \) Definition of congruent triangles.

Exercise 6.2

In the figures shown below, \( \triangle LCM \) and \( \triangle BKJ \) are congruent triangles. Complete the following congruence statements.

1. \( L \cong ________ \)

2. \( \angle JK \cong ________ \)

3. \( \triangle CML \cong ________ \)

4. \( K \cong ________ \)

5. \( \angle LMC \cong ________ \)

6. \( \triangle KBJ \cong ________ \)

6.3 Tests for Congruency of Triangles

How do you check the congruency of triangles? From the previous section you know that, to say one triangle is congruent to the other, one has to check the congruency of all the corresponding sides and all the corresponding angles. However, you can check the congruency of two triangles using some short cut techniques. These techniques are named as **congruence tests**.

Activity 6.3

1. Construct two triangles with congruent corresponding sides as shown below.

2. Using protractor measure each angles of the triangle and compare the corresponding angle measures.

3. Do the above steps for another pair of triangles with congruent corresponding sides.
4. What do you conclude about congruency of two triangles with given only corresponding sides congruent?

Side-Side-Side (SSS) Congruency Test

If three sides of one triangle are congruent to the corresponding three sides of another triangle, then the two triangles are congruent. That is:

If \( AB \cong DE \), \( BC \cong EF \), \( AC \cong DF \)

Then \( \triangle ABC \cong \triangle DEF \)

Example 6.6

In the figure to the right, let \( AB = DB = 8 \text{cm} \) and \( AC = DC = 5 \text{cm} \).

Show that \( \triangle ABC \cong \triangle DBC \)

Solution:

\( AB \cong DB \), given
\( AC \cong DC \), given
\( BC \), common side

Hence, \( \triangle ABC \cong \triangle DBC \) by SSS congruence test

Example 6.7

Suppose that \( ABCD \) is a kite, show that the longer diagonal \( AC \) divides the kite into two congruent triangles

Solution:

i. \( AB \cong AD \)  
   Definition of kite
ii. \( CB \cong CD \)  
   Definition of kite
iii. \( AC \cong AC \),  
    Common side
Thus, all the three pairs of corresponding sides of $\triangle ABC$ and $\triangle ADC$ are congruent.

Therefore, $\triangle ABC \cong \triangle ADC$ by SSS congruence test

**Example 6.8**

In the figure to the right, $\overline{EF} \cong \overline{JI}$, $\overline{FG} \cong \overline{IH}$ and $m(\angle EFG)=m(\angle JIH)=90^\circ$, then show that $\triangle EFG$ and $\triangle JIH$ are congruent.

**Solution:**

a. $\overline{EF} \cong \overline{JI}$, given

b. $\overline{FG} \cong \overline{IH}$, given

c. $\overline{EG} \cong \overline{JH}$ hypotenuse of two right triangles with corresponding legs equal

Therefore, $\triangle EFG \cong \triangle JIH$ by SSS congruence test

**Activity 6.4**

1. Construct a triangle with two sides and the included angle is congruent with the corresponding two sides and the included angle of another triangle, as shown in the figure below.

2. Using protractor and ruler measure the remaining angles and side lengths. Compare the measure of the corresponding angles and sides.

3. Do the above steps for another pair of triangles

4. What do you conclude about congruency of the two triangles with given only two pair of corresponding sides and included angles congruent?

**Side-Angle-Side (SAS) Congruency Test**

If two sides and the included angle of one triangle are congruent to the corresponding two sides and the included angle of another triangle, then the two triangles are congruent.
That is:
If \( \overline{AB} \cong \overline{DE}, \overline{BC} \cong \overline{EF} \) and \( \angle ABC \cong \angle DEF \), then \( \triangle ABC \cong \triangle DEF \)

**Example 6.9**

Look at the figure to the right. Given \( \overline{AB} \cong \overline{AC} \) and \( \angle BAP \cong \angle CAP \). Show that \( \triangle BAP \cong \triangle CAP \).

**Solution:**

i. \( \overline{AB} \cong \overline{AC} \)

ii. \( \angle BAP \cong \angle CAP \), given

iii. \( \overline{AP} \), common side

Hence, \( \triangle BAP \cong \triangle CAP \), by the SAS congruence test

**Example 6.10**

Given \( \overline{RT} \cong \overline{UT} \) and \( \overline{ST} \cong \overline{VT} \), and as in the figure below, show that \( \triangle RTS \cong \triangle UTV \).

**Solution:**

a. \( \overline{ST} \cong \overline{VT} \), given,

b. \( \overline{RT} \cong \overline{UT} \), given.

c. \( \angle RTS \cong \angle UTV \), vertically opposite angles

Therefore, \( \triangle RTS \cong \triangle UTV \), by SAS congruence test.

**Example 6.11**

Given \( \triangle AFB \) and \( \triangle CED \) as shown on the following dot paper (Dots with equal distance), show that the sides \( \overline{AB} \) and \( \overline{CD} \) are congruent
Solution:

a. \( \overline{FB} \cong \overline{ED} \), given as 3 units (joining 4 dots)

b. \( \overline{FA} \cong \overline{EC} \), given as 1 unit (joining 2 dots)

c. \( \angle AFB \cong \angle CED \), right angles

Then, \( \triangle AFB \cong \triangle CED \), by SAS congruence test

Therefore, \( \overline{AB} \cong \overline{CD} \) corresponding sides of two congruent triangles

Angle-Side-Angle (ASA) Congruency Test

If two angles and the included side of one triangle are congruent to the corresponding two angles and the included side of another triangle, then the two triangles are congruent.

That is; If \( \angle BAC \cong \angle EDF \), \( \angle ACB \cong \angle DFE \), \( \overline{AC} \cong \overline{DF} \) then \( \triangle ABC \cong \triangle DEF \)

Example 6.12

In the figure below, \( \overline{AC} \cong \overline{BC} \) and \( \angle ACD \cong \angle BCE \), then show that \( \triangle ACD \cong \triangle BCE \)

Solution:

i. \( \overline{AC} \cong \overline{BC} \), given.

ii. \( \angle ACD \cong \angle BCE \), given

iii. \( \angle CBE \cong \angle CAD \), base angles of isosceles \( \triangle ACB \)

Thus, \( \triangle ACD \cong \triangle BCE \), by the ASA congruence test.
Example 6.13

In the figure below, let \( m(\angle FBS) = m(\angle MTS) = 90^\circ \) and \( \overline{SB} \cong \overline{ST} \), then show \( \triangle FSB \cong \triangle MST \).

Solution:

i. \( \overline{SB} \cong \overline{ST} \) given

ii. \( \angle FBS \cong \angle MTS \) right angles

iii. \( \angle BSF \cong \angle TSM \) vertical opposite angles

Thus, \( \triangle FSB \cong \triangle MST \) by the ASA congruence test.

Activity 6.5

1. Construct a pair of right triangles with hypotenuse and one side are congruent with the corresponding hypotenuse and one side of another right triangle.

2. Using protractor and ruler measure the remaining angles and side lengths. Compare the measure of the corresponding angles and sides.

3. Construct another pair of triangles and repeat the activity.

4. What do you conclude about the congruency of the two right triangles with given only their hypotenuse and one corresponding side congruent?

Right Angle – Hypotheses – Side (RHS) Congruency Test

If two right angled triangles have one pair of congruent sides (legs) and congruent hypotenuses, then the triangles are congruent. That is:

If \( \angle CAB \) and \( \angle FDE \) are right angles, \( \overline{AC} \cong \overline{DF} \), and hypotenuse \( \overline{BC} \cong \overline{EF} \), then \( \triangle ABC \cong \triangle DEF \) by RHS test.

Example 6.14

Show that the altitude of an isosceles triangle divides the triangle into two congruent right triangles.
**Solution:**

Let $\overline{AE}$ be the altitude of an isosceles triangle $ABC$

i. $\overline{AB} \cong \overline{AC}$, congruent sides of an isosceles triangle

ii. $\overline{AE}$, Common side

iii. $m(\angle AEB) = m(\angle AEC) = 90^\circ$, $\overline{AE}$ is the altitude of $\triangle ABC$

Thus, $\angle AEB \cong \angle AEC$

Therefore, $\triangle ABE \cong \triangle ACE$ by RHS congruence test

---

**Example 6.15**

Given, $\overline{AB} \perp \overline{BC}$, $\overline{DE} \perp \overline{EF}$, $\overline{AC} \cong \overline{DF}$, $\overline{BF} \cong \overline{EC}$. Show that $\triangle ABC \cong \triangle DEF$

**Solution:**

i. $\overline{AC} \cong \overline{DF}$, given

ii. $\overline{BF} \cong \overline{EC}$, given

iii. $\overline{BC} \cong \overline{EF}$, why?

iv. $m(\angle ABC) = m(\angle DEF) = 90^\circ$, given

Therefore, $\triangle ABC \cong \triangle DEF$, by RHS congruence test

---

**Exercise 6.3**

1. The figure $ABCD$ is a rhombus where diagonals $\overline{AC}$ and $\overline{BD}$ intersects at $O$. Show that
   a. $\triangle BOC \cong \triangle DOA$
   b. $\triangle BOA \cong \triangle DOA$

2. In the figure below, if $m(\angle RPQ) = m(\angle UST) = 40^\circ$, $m(\angle PQR) = m(\angle STU) = 60^\circ$ and $RQ = UT = 10$ cm, then show that $\triangle PQR$ and $\triangle STU$ are congruent?
3. Decide which condition is missed for the following pairs of triangles to be congruent. Explain.

   a) A B C D E
      A C B
   b) A B C D E F
      A D B F
   c) A E C B
      D E C
   d) A E F B H
      C F J K

4. Asnake says these two triangles must be congruent. Do you agree? Why?

   L O M N
   P Q

5. In the triangles below, the angle measures are given as indicated. Can we say \( \triangle ABC \) is congruent to \( \triangle XYZ \)? Why?

   A B C
   Z X

6.4 Introduction to Similarity of Triangles

   Activity 6.6

   1. The following pairs of plane figures are not necessarily congruent. What is missing for each to be congruent?
      a. Two equilateral triangles.
      b. Two circles.
      c. Two squares

   2. In the figures given, all the corresponding angles of \( \triangle ABC \) and \( \triangle DEF \) are congruent to each other.
However, the corresponding sides are not congruent; the lengths of the sides are given as indicated in the figure.

a. Is \( \triangle ABD \cong \triangle DEF \)?

b. Compare the ratio of the corresponding sides.

c. What can you say about the two triangles?

In geometry, there are plane figures having all the corresponding angles congruent but the corresponding sides may not be congruent instead the sides are proportional. Such plane figures in which one is the enlarged or reduced form of the other are called \textit{similar plane figures}.

\textbf{Definition:}

\( \triangle ABC \) is similar to \( \triangle DEF \), written as \( \triangle ABC \sim \triangle DEF \), if and only if, the corresponding interior angles are congruent and the lengths of the corresponding sides are proportional. That is:

\[
\begin{align*}
\angle BAC & \cong \angle EDF \\
\angle ABC & \cong \angle DEF \\
\angle ACB & \cong \angle DFE
\end{align*}
\]

\( \frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DE} \) corresponding proportional sides

Therefore, \( \triangle ABC \sim \triangle DEF \)

\textbf{Example 6.16}

In the figure below, show that \( \triangle DEF \) and \( \triangle XYZ \) are similar

\textbf{Solution:}

i. \( m(\angle XYZ) = m(\angle DEF) = 82^\circ \), given

\( m(\angle YXZ) = m(\angle EDF) = 70^\circ \), given

\( m(\angle YZX) = m(\angle EFD) = 28^\circ \), given

The corresponding angles are congruent.
ii. \( \frac{YX}{ED} = \frac{30\text{cm}}{20\text{cm}} = \frac{3}{2} \)
\( \frac{XZ}{DF} = \frac{60\text{cm}}{40\text{cm}} = \frac{3}{2} \)
\( \frac{YZ}{EF} = \frac{45\text{cm}}{30\text{cm}} = \frac{3}{2} \)

This implies that the lengths of corresponding sides are in equal ratio. That is the corresponding sides are proportional. Therefore, by (i) and (ii), \( \triangle DBF \sim \triangle XYZ \)

**Example 6.17**

Suppose that \( \triangle ABC \sim \triangle ADE \) with \( AB = 11\text{cm} \), \( BC = 8\text{cm} \), \( AE = 4\text{cm} \) and \( DE = 3\text{cm} \). Find the length of the sides, \( AD \) and \( AC \).

**Solution:**

Since, \( \triangle ABC \sim \triangle ADE \) we have

\[
\frac{AD}{AB} = \frac{DE}{BC} = \frac{AE}{AC} \\
\text{Proportionality of corresponding sides}
\]

\[
\frac{AD}{11\text{cm}} = \frac{3\text{cm}}{8\text{cm}}
\]

Substitution

\[
AD = 11\text{cm} \times \frac{3\text{cm}}{8\text{cm}} = \frac{33\text{cm}}{8} ; \text{ and}
\]

\[
\frac{3\text{cm}}{8\text{cm}} = \frac{4\text{cm}}{AC}
\]

\[
AC = \frac{4\text{cm} \times 8\text{cm}}{3\text{cm}} = \frac{32\text{cm}}{3}
\]

Like similar triangles, any two polygons are similar, if and only if, each of the corresponding pairs of interior angles are congruent and the corresponding sides are proportional.

**Exercise 6.4**

1. Given that \( \triangle ABC \) and \( \triangle ADE \) are similar triangles, as shown in the figure.
   a. Show the congruence of the corresponding angles,
   b. Show the proportionality of the corresponding sides.
2. As shown in the figure, \(AH = 12\text{cm}, AK = 9\text{cm}, HK = 15\text{cm}, HE = 5\text{cm}, HG = 4\text{cm}, GE = 3\text{cm}, m(\angle KAH) = 90^\circ\) and \(m(\angle EGH) = 90^\circ\). Justify that \(\triangle KAH \sim \triangle EGH\).

![Diagram](image1)

3. In the figure below given that, \(\triangle ABC \sim \triangle DEF\), with \(AB=3\text{cm}, BC=12\text{cm}, FD=3.5\text{cm}, DE=1\text{cm}\) and \(FE=4\text{cm}\). Find the missing length \(AC\).

![Diagram](image2)

4. In the figures below if \(\square PQRS \cong \square LMNT\) and \(\triangle ABC \cong \triangle DEF\). Find the value of \(x\)

![Diagram](image3)

**Problem Solving**

A group of engineers formed the triangle with the measurement \(AS=8\text{m}, CR=14\text{m}, SD=4\text{m}, RD=7\text{m},\) and \(m(\angle ASD)= m(\angle CRD)= 90^\circ\) as shown in the figure. Is \(\triangle ASD\) similar with \(\triangle CRD\)? If so justify your reason.
UNIT 6: CONGRUENCY OF PLANE FIGURES

Unit Summary

❖ Any geometric figures having the same size and shape are congruent.
❖ If the corresponding parts of two triangles are congruent then the two triangles are congruent and the vice versa.
❖ Congruency of triangles can be tested by SSS, SAS, ASA, and RHS congruency tests.
❖ Similar triangles have the same shape but may not have the same size.
❖ Any two congruent triangles are similar.
❖ Similarity is not the necessary condition for congruency.

Review Exercises

1. If $\Delta ABC \cong \Delta DEF$, list the three pairs of corresponding congruent sides and the three pairs of corresponding congruent angles.

2. Determine whether or not the given information is sufficient to conclude that the triangles are congruent by congruence tests. If congruent, state the test that shows the triangles are congruent. If not necessarily congruent explain why not.

3. Given $\overline{AC} \cong \overline{BC}$ and $\angle A \cong \angle B$ show that $\Delta AEC \cong \Delta BDC$.
4. In the figure below, given \( LM = ON \) and \( LN = OM \), show that \( \triangle LMN \cong \triangle ONM \)

5. In the figure at the right, \( \angle WVU \cong \angle RST \), \( \overline{VU} \cong \overline{ST} \) and \( \overline{PS} \cong \overline{OV} \). Identify three pairs of congruent triangles

6. For each pair of triangles, tell whether the given information is enough to show that the triangles are congruent. If the triangles are congruent, state the criterion that you used to determine the congruency.

7. If \( \triangle ABC \cong \triangle DEF \), find the measures of the given angles or the lengths of the given sides when.
   a. \( m(\angle A) = x + 10 \), and \( m(\angle D) = 2x \)
   b. \( m(\angle B) = 3y \), and \( m(\angle E) = 6y - 12 \)
   c. \( BC = 3z + 2 \), and \( EF = z + 6 \)
   d. \( AC = 7a + 5 \), and \( DF = 5a + 9 \)

8. In the figure \( PQRS \) is a rectangle and \( PQT \) is an equilateral triangle. Show that \( \triangle PSM \cong \triangle QRN \)
9. In the figure below, given that \(\overline{DO} \cong \overline{BO}\) and \(\angle ADO \cong \angle CBO\) and points D, O and C are on a straight line. Show that, \(\triangle AOD \cong \triangle COD\).

![Diagram of triangle AOD and COD](image)

10. In the figure, given \(\overline{AC} \cong \overline{DE}, \overline{AB} \cong \overline{EO}\) and \(\angle ABC \cong \angle EOD\), show that \(\triangle ABC \cong \triangle EOD\).

![Diagram of triangle ABC and EOD](image)

11. In the rectangle \(ABCD\) shown, \(X\) and \(Y\) are midpoints of the given sides and \(DP = AQ\).
   
   a. What type of quadrilateral is \(PYQX\)? Prove your answer.
   
   b. If points \(P\) and \(Q\) are moved at a constant rate and in the same direction along \(DC\) and \(AB\), respectively, does this change your answer in part (a)? Why or why not?

![Diagram of rectangle ABCD with midpoints X and Y](image)
UNIT - 7
DATA HANDLING

Learning Outcomes:

After completing this unit, you will be able to:
- Organize data using frequency tables
- Construct and interpret pie chart
- Calculate Mean, Mode, Median and Range of a given data
- Apply the concept of data handling to organize and interpret real life problems

Key Terms
- Data
- Discrete data
- Value
- Tally marks
- Line graph
- Pie chart
- Frequency
- Proportion
- Median
- Mean
- Mode
- Range
INTRODUCTION

Collecting data about your surroundings and representing systematically using pictures or graphs and describing the whole and parts of the data will help you to clearly understand the surrounding and enable you make effective decisions in life.

In this unit, you will learn how to collect simple data, organize a given discrete data in tally marks and frequency tables, and represent the data using pie charts and line graphs. You will also compute the mean, median, mode and ranges of a given data.

### 7.1 Organization of Data Using Frequency Table

#### Activity 7.1

Decide whether each of the following statement is correct or not. To say correct or not you have to collect, organize and analyze data about cases in your class.

- a. The number of boys in your class is greater than the number of girls.
- b. More than half of the students in your class know how to use a calculator.
- c. 5 students are less than 13, 12 students are 14, and 9 students are 15 years old.
- d. Half of the students in your class are 1.5 meters tall.
- e. 13 students in your class know how much water the Renaissance Dam holds.
- f. There are 7 students in your class who write with their left hand.

#### How data can be collected?

Data can be collected using questionnaires, by making observations and recording the results, by carrying out experiments, from records or databases, from the internet and the like.

Collecting data requires preparing statistical questions that anticipates variability. For example, “How old am I?” is not a statistical question, but “How old are the students in my school?” is a statistical question because one anticipates variability in students’ ages.

#### Activity 7.2

1. Copy the following and fill each by putting a tick mark (✓) in only one of the blank spaces given (individual work)
   - a. Sex: Male ______ Female ______
   - b. Age: Under 12 ___ 12 to 13 years____ 14 and above ____.
   - c. Which fruit do you like most?
     - Orange ______ Banana ______ Mango ______ Avocado ______

2. Collect the papers and organize the data in a way that everyone can easily understand (Whole class work).
From the activity you have done, how many students are found to be males? How many are females? How many of you are in different age categories? How many of you like different types of fruits? Answering these questions requires data representation and organization skills.

Data can be organized in the form of frequency table and represented with tally marks. Tally marks are grouped in five to make them easier to count. For example, \(\begin{array}{cccc} x & x & x & x & x \end{array} \) is easier to count than \(\begin{array}{cccccc} x & x & x & x & x & x & x & x & x \end{array} \). The tally in a form \(\begin{array}{cc} x & x \end{array} \) represents five members of the group.

### Example 7.1

60 grade seven students read different types of fiction books. The number of books they read is given to the right.

<table>
<thead>
<tr>
<th>Books Read</th>
<th>Tally</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>(\begin{array}{cc} x &amp; x \end{array} )</td>
<td>6</td>
</tr>
<tr>
<td>1</td>
<td>(\begin{array}{ccc} x &amp; x &amp; x &amp; x &amp; x \end{array} )</td>
<td>12</td>
</tr>
<tr>
<td>2</td>
<td>(\begin{array}{cccccc} x &amp; x &amp; x &amp; x &amp; x &amp; x &amp; x &amp; x \end{array} )</td>
<td>19</td>
</tr>
<tr>
<td>3</td>
<td>(\begin{array}{ccc} x &amp; x &amp; x &amp; x &amp; x &amp; x &amp; x &amp; x &amp; x \end{array} )</td>
<td>8</td>
</tr>
<tr>
<td>4</td>
<td>(\begin{array}{ccc} x &amp; x &amp; x &amp; x &amp; x &amp; x &amp; x &amp; x &amp; x \end{array} )</td>
<td>8</td>
</tr>
<tr>
<td>5</td>
<td>(\begin{array}{cccc} x &amp; x &amp; x &amp; x &amp; x &amp; x \end{array} )</td>
<td>4</td>
</tr>
<tr>
<td>6</td>
<td>(\begin{array}{ccc} x &amp; x &amp; x &amp; x &amp; x &amp; x &amp; x \end{array} )</td>
<td>3</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td></td>
<td><strong>60</strong></td>
</tr>
</tbody>
</table>

Based on the given information

a. Make a table and organize the data.

b. Answer the following questions

i. How many students read five books and more?

ii. How many students read three or less number of fiction books?

iii. How many students didn’t read a fiction book?

**Solution:**

a.

- \(4 + 3 = 7\) Students read five books and more.
- \(8 + 19 + 12 + 6 = 45\) Students read three or less number of fiction books
- \(6\) students didn’t read a fiction book.
Exercise 7.1

3. Consider the following data collected from the scores of 40 sample students in a mathematics examination. Arrange the data with tally marks and make a frequency table.

| 56 | 78 | 62 | 37 | 54 | 39 | 62 | 60 |
| 28 | 82 | 38 | 72 | 62 | 44 | 54 | 42 |
| 42 | 55 | 57 | 65 | 68 | 47 | 42 | 56 |
| 56 | 56 | 55 | 66 | 42 | 52 | 48 | 48 |
| 47 | 41 | 50 | 52 | 47 | 48 | 53 | 68 |

a. How many students obtained marks greater than or equal to 65?
b. How many students obtained marks below 50?

4. The following are the weights (in kg.) of 22 peoples. Arrange the data in frequency table.

| 60 | 30 | 50 | 20 | 40 | 30 |
| 60 | 20 | 60 | 30 | 60 | 20 |
| 40 | 30 | 60 | 20 | 30 | 20 |
| 50 | 40 | 30 | 30 |

5. The daily sales (in Birr) of 20 shops in some town of Amhara Region is given as follows. Organize the given data in tally marks and frequencies.

| 2230 | 2215 | 2225 | 2230 | 2225 | 2220 | 2230 | 2225 |
| 2220 | 2225 | 2220 | 2225 | 2215 | 2230 | 2225 | 2215 |
| 2220 | 2225 | 2230 | 2220 |

ProblemSolving

Project Work: Collect information regarding the number of family members of students in your class. Copy and use the table below to make a frequency table.

<table>
<thead>
<tr>
<th>Number of family members</th>
<th>Tally marks</th>
<th>Number of students with that number of family</th>
</tr>
</thead>
<tbody>
<tr>
<td>2 and less than 2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6 and more than 6</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
7.2 Construction and Interpretation of Pie Charts and Line Graphs

In the previous section, you have seen how a data is collected, organized and presented in frequency table. This data can be transferred to a chart or a graph form. There is a connection between data in tables and data in charts or graphs. Can you say something about these connections?

One of the common types of charts we used to represent data is Pie chart. Pie Chart is a circular graph that is divided into sections that are proportional to the data they represent. It gets its name from its appearance, which resembles a pie (Circular bread) that has been cut into different sized slices.

Pie chart is a common and accurate way of representing data especially useful for showing the relations of one item with another and one item with the whole items.

Activity 7.3

The pie chart below shows the number of visitors coming to visit Gorgora Park each day in a week.

Look at the pie chart carefully and discuss what the chart is showing

a. What percent of visitors attended on each day of the week?
b. In which day large number of visitors visited Gorgora?
c. On which day is the number of visitors smaller?
d. How many visitors attended on each day of the week?
UNIT 7: DATA HANDLING

Pie chart helps to present data in simple and understandable way even to those who are not familiar to mathematical figures. The observer gets the information at a glance, no need to spend a lot of time in studying the chart. In addition, pie charts are pleasing, and easily attract people to the basic information.

Each sector of the pie chart should be labeled, so that one can associate the section with the data it represents. Usually, the different sections of a pie chart are presented in different colors or shades, so that they can easily be differentiated.

Pie chart is one way of representing data. A circle is divided into sectors. The total data is represented by the circular region as a whole and the individual data by the sector of the circle. The larger the angle of the sector at the center of the circle, the larger its frequency.

Example 7.2

Suppose 40 students were asked to identify their favorite color. The data is as given below. Construct a pie chart for the data.

<table>
<thead>
<tr>
<th>Blue</th>
<th>Green</th>
<th>White</th>
<th>Blue</th>
<th>Black</th>
<th>Green</th>
<th>Yellow</th>
<th>Green</th>
<th>Black</th>
<th>Yellow</th>
</tr>
</thead>
<tbody>
<tr>
<td>Yellow</td>
<td>Orange</td>
<td>White</td>
<td>Black</td>
<td>Blue</td>
<td>Yellow</td>
<td>Green</td>
<td>Black</td>
<td>Yellow</td>
<td>Blue</td>
</tr>
<tr>
<td>Red</td>
<td>Yellow</td>
<td>Blue</td>
<td>Green</td>
<td>White</td>
<td>Blue</td>
<td>Orange</td>
<td>Green</td>
<td>Orange</td>
<td>Green</td>
</tr>
<tr>
<td>Yellow</td>
<td>Yellow</td>
<td>Black</td>
<td>Red</td>
<td>Black</td>
<td>Yellow</td>
<td>Yellow</td>
<td>white</td>
<td>Green</td>
<td>Yellow</td>
</tr>
</tbody>
</table>

Solution:

Step 1: Represent the data in the form of a frequency distribution table.

<table>
<thead>
<tr>
<th>Color Choice</th>
<th>Tally</th>
<th>Frequency (Number of students)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Blue</td>
<td>//</td>
<td>6</td>
</tr>
<tr>
<td>Red</td>
<td>//</td>
<td>2</td>
</tr>
<tr>
<td>Green</td>
<td>//</td>
<td>8</td>
</tr>
<tr>
<td>Yellow</td>
<td>//</td>
<td>11</td>
</tr>
<tr>
<td>Others</td>
<td>//</td>
<td>13</td>
</tr>
<tr>
<td></td>
<td>Total</td>
<td>60</td>
</tr>
</tbody>
</table>

Step 2: Determine the proportion of each color choice in percent.

\[
\text{Percent of color choice} = \frac{\text{Number of one Color choice}}{\text{Total number of participants}} \times 100\%
\]
### Step 3: Determine the proportion of each color choice in degree measure.

Degree measure of one sector = \( \frac{\text{Number of one Color choice}}{\text{Total number of participants}} \times 360^\circ \)

<table>
<thead>
<tr>
<th>Choice of color</th>
<th>Number of students</th>
<th>Sector representing the choice</th>
</tr>
</thead>
<tbody>
<tr>
<td>Blue</td>
<td>6</td>
<td>( \frac{6}{40} \times 360^\circ = 54^\circ )</td>
</tr>
<tr>
<td>Red</td>
<td>2</td>
<td>( \frac{2}{40} \times 360^\circ = 18^\circ )</td>
</tr>
<tr>
<td>Green</td>
<td>8</td>
<td>( \frac{8}{40} \times 360^\circ = 72^\circ )</td>
</tr>
<tr>
<td>Yellow</td>
<td>11</td>
<td>( \frac{11}{40} \times 360^\circ = 99^\circ )</td>
</tr>
<tr>
<td>Others</td>
<td>13</td>
<td>( \frac{13}{40} \times 360^\circ = 117^\circ )</td>
</tr>
</tbody>
</table>

### Step 4: Construct a circle and using the degree proportions divide the circle into sectors. Use protractor to determine the size of each sector.
UNIT 7: DATA HANDLING

Remark:

The steps to construct the pie chart can be summarized as:

1. Arrange the raw data in frequency distribution table
2. Express each section of the data in percent
3. Determine the proportion of each item in degree measure.
4. Construct a circle using compasses
5. Using a protractor divide the circle into sectors with the respective degree measure proportions of each item.

Key Features of a Pie Chart

- A circle is used to represent all the data
- Must have a title
- Each sector must be labeled, or a key should be provided to aid interpretation
- Each sector is calculated as a part of the whole
- When comparing pie charts, the total sample size should be provided

Example 7.3

The pie chart shows the number of teachers in a certain school within four departments: Language, Mathematics, Natural science and Social science.

If the total number of teachers is 48, then how many mathematics teachers are there in the school?

Solution:

i. Let the unknown degree measure be \( x \)

Angle of the sectors representing Language + Social science + Natural science + Mathematics = 360°

\[
135° + 60° + 75° + x = 360°
\]

\[
270° + x = 360°
\]

\[
x = 360° - 270° = 90°
\]
ii. Find the number of mathematics teachers.

Degree measure for mathematics = \(\frac{\text{number of Mathematics teachers}}{\text{total number of teachers}} \times 360^\circ\)

\(90^\circ = \frac{\text{number of Mathematics teachers}}{48} \times 360^\circ\)

Number of mathematics teachers = \(\frac{90^\circ \times 48}{360^\circ} = \frac{48}{4} = 12\)

Therefore, there were 12 mathematics teachers in the school.

**Exercise 7.2**

1. The pie chart below shows how long a gardener spent doing various activities over 20 days.
   a. What proportion of the time was spent for each of the activities?
   b. For which activity the gardener used more time?

2. The ages of 48 students in a class were recorded as follows. Construct a pie chart that represents the data.

<table>
<thead>
<tr>
<th>14</th>
<th>14</th>
<th>14</th>
<th>14</th>
<th>16</th>
<th>14</th>
<th>14</th>
<th>16</th>
<th>14</th>
<th>15</th>
<th>14</th>
</tr>
</thead>
<tbody>
<tr>
<td>14</td>
<td>15</td>
<td>17</td>
<td>15</td>
<td>13</td>
<td>14</td>
<td>15</td>
<td>16</td>
<td>13</td>
<td>15</td>
<td>13</td>
</tr>
<tr>
<td>17</td>
<td>14</td>
<td>13</td>
<td>14</td>
<td>15</td>
<td>15</td>
<td>17</td>
<td>16</td>
<td>15</td>
<td>16</td>
<td>15</td>
</tr>
<tr>
<td>13</td>
<td>14</td>
<td>14</td>
<td>16</td>
<td>14</td>
<td>14</td>
<td>15</td>
<td>13</td>
<td>16</td>
<td>14</td>
<td>16</td>
</tr>
</tbody>
</table>

3. The following pie chart shows different ways that students travel to school. Look at the pie chart to answer the question that follows.
   a. What percent of students come to school on foot?
   b. What percent of students use transport services?
   c. If the total number of students is 2000, how many students come to school on foot? How many students use Bajaj?
UNIT 7: DATA HANDLING

Line Graph

Another method of presenting data visually is the line graph. A line graph is a sequence of points connected by line segments and is often used to show changes over a period of time. For example, the line graph below shows the increase in number of students in a school from 2000 to 2013 years of interval.

![Line Graph](image)

**Activity 7.4**

Use the line graph above to answer the following questions.

a. What was the number of students in 2008?

b. What was the approximate number of students increase from 2008 to 2009?

c. In which year interval was the decrease in the number of students observed?

d. Estimate and complete the following table by looking the line graph

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Students in school</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The line graph is most commonly used to represent two related facts. To construct a line graph, for a given data you can use your knowledge of drawing a line on a Cartesian coordinate plane.
Example 7.4

A car uses 1 liter of petrol for every 10 km it travels

a. Copy and complete the table below showing how much liter of petrol the car uses.

<table>
<thead>
<tr>
<th>Distance travelled in km</th>
<th>0</th>
<th>10</th>
<th>20</th>
<th>30</th>
<th>40</th>
<th>50</th>
<th>60</th>
</tr>
</thead>
<tbody>
<tr>
<td>Petrol used in liters</td>
<td>0</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

b. Draw a line graph using the information in the above table.

c. How much liters of petrol did the car use to travel 7km?

d. How many kilometers did the car travel with 10 liters of petrol?

Solution:

a. Let L be the number of liters the car consumes for a given km, and;

Let K be the distance covered by the car.

Distance (K) = 10km/liter \times L

When distance = 40km, 40km=10km / liter \times L

Thus, \( L = \frac{40\text{ km}}{10\text{ km/liter}} = 4\text{ liters} \)

When distance = 50kms, 50km=10km / liter \times L

\( L = \frac{50\text{ km}}{10\text{ km/liter}} = 5\text{ liters} \)

When distance = 60kms, 60km=10km / liter \times L

\( L = \frac{60\text{ km}}{10\text{ km/liter}} = 6\text{ liters} \)

Therefore, the complete frequency table is:

<table>
<thead>
<tr>
<th>Distance travelled in km</th>
<th>0</th>
<th>10</th>
<th>20</th>
<th>30</th>
<th>40</th>
<th>50</th>
<th>60</th>
</tr>
</thead>
<tbody>
<tr>
<td>Petrol used in liters</td>
<td>0</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>6</td>
</tr>
</tbody>
</table>

b. To draw the graph, plot the points (0, 0), (10, 1), (20, 2), (30, 3), (40, 4), (50, 5) and (60, 6) on xy-plane.
UNIT 7: DATA HANDLING

c. For 7 km, number of liters is:
\[
\frac{(7 \text{ km}) \times 1\text{Liter}}{10 \text{ km}} = 0.7 \text{ Liters}
\]
d. \(K=10\text{km/litter}\times L\)
\[=10\text{km/liter}\times10\text{liters}=100\text{km}\]

Exercise 7.3

1. Draw line graphs to represent each of the following data.
   a. The number of letters delivered to an office in one week
   
<table>
<thead>
<tr>
<th>Weeks</th>
<th>Sat</th>
<th>Sun</th>
<th>Mon</th>
<th>Tue</th>
<th>Wed</th>
<th>Thu</th>
<th>Fri</th>
</tr>
</thead>
<tbody>
<tr>
<td>Letters</td>
<td>20</td>
<td>0</td>
<td>12</td>
<td>25</td>
<td>15</td>
<td>19</td>
<td>23</td>
</tr>
</tbody>
</table>

   b. The temperature in Addis Ababa at midday during the first week in July
   
<table>
<thead>
<tr>
<th>Day</th>
<th>Sat</th>
<th>Sun</th>
<th>Mon</th>
<th>Tue</th>
<th>Wed</th>
<th>Thu</th>
<th>Fri</th>
</tr>
</thead>
<tbody>
<tr>
<td>Temperature (°C)</td>
<td>12</td>
<td>16</td>
<td>14</td>
<td>11</td>
<td>12</td>
<td>15</td>
<td>13</td>
</tr>
</tbody>
</table>

2. The depth of water in a reservoir is 144m. During a dry period, the water level falls by 4m each week.
   a. Copy and complete the table below which shows the variation of depth of water in the reservoir.

<table>
<thead>
<tr>
<th>Week</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Expected depth of water in meter(m)</td>
<td>144</td>
<td>140</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

   b. Draw a line graph using the information in the table above.
   c. How deep would you expect the level of the water to be after 10 weeks?
   d. If the water level falls to 96m the water company will divert water from another reservoir. After how long will the company make the diversion?

Activity 7.5

How can we use Microsoft excel to construct a line graph and pie chart for a given data?

A computer spreadsheet (excel) is useful to construct pie chart and line graphs.
Example 7.5

The table gives the number of grade 7 students in a school that scored above 60% in mathematics examination.

<table>
<thead>
<tr>
<th>Mark out of 100</th>
<th>91-100</th>
<th>81-90</th>
<th>71-80</th>
<th>61-70</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Students</td>
<td>17</td>
<td>33</td>
<td>52</td>
<td>86</td>
</tr>
</tbody>
</table>

**Solution:**

To construct a pie chart of the data, follow these steps

Step 1: Change the number of students into percentage

<table>
<thead>
<tr>
<th>Mark out of 100</th>
<th>Number of Students</th>
<th>Percentage</th>
</tr>
</thead>
<tbody>
<tr>
<td>91-100</td>
<td>17</td>
<td>9.04</td>
</tr>
<tr>
<td>81-90</td>
<td>33</td>
<td>17.55</td>
</tr>
<tr>
<td>71-80</td>
<td>52</td>
<td>27.66</td>
</tr>
<tr>
<td>61-70</td>
<td>86</td>
<td>45.74</td>
</tr>
</tbody>
</table>

Step 2: To make a circle graph or pie chart, highlight the data in A1 through B4.

Step 3: Click on insert, then and choose Pie.

Step 4: Click Next to enter the chart title. Then click Next and Finish.
UNIT 7: DATA HANDLING

To construct a line graph of the data, follow these steps.

Step 1: Set up a spreadsheet, with the mark out of 100 in column A and the number of students in column B.

Step 2: highlight the data in A1 through B4.

Step 3: Click on the Chart Wizard icon, choose the line graph.

Step 4: To set up axis titles, choose the layout tab and press the axis titles icon.

Step 5: Then choose primary horizontal or primary vertical axis titles.

Step 6: Give the title names to each axis.

Exercise 7.4

1. The table shows the results of a survey in which students were asked to indicate their favorite type of magazine. Use a spreadsheet to make a pie chart of these data.

<table>
<thead>
<tr>
<th>Comics</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fashion</td>
<td>7</td>
</tr>
<tr>
<td>Entertainment</td>
<td>5</td>
</tr>
<tr>
<td>News</td>
<td>3</td>
</tr>
<tr>
<td>Sports</td>
<td>6</td>
</tr>
<tr>
<td>Other</td>
<td>1</td>
</tr>
</tbody>
</table>

2. Abebe and his family often watch movies at home. The data shows the number of movies watched by them from 2008 to 2012. Use a spreadsheet to draw a line graph to represent the data.

3. COLLECT THE DATA: Collect some data that can be displayed in a pie chart. Record the data in a spreadsheet. Then use the spreadsheet to make the pie chart display. Is the display appropriate? Justify your answer.

<table>
<thead>
<tr>
<th>Year</th>
<th>Number of Movies</th>
</tr>
</thead>
<tbody>
<tr>
<td>2008</td>
<td>8</td>
</tr>
<tr>
<td>2009</td>
<td>12</td>
</tr>
<tr>
<td>2010</td>
<td>10</td>
</tr>
<tr>
<td>2011</td>
<td>14</td>
</tr>
<tr>
<td>2012</td>
<td>18</td>
</tr>
</tbody>
</table>
7.3 Mean, Mode, Median and Range of Data

In this sub-unit, you will learn about mean, median, mode and range of the given data. To compute the values of these measures, you can use raw data or data organized in the form of frequency tables.

**Activity 7.6**

Recall and list your second semester grade 6 final examination marks of all subjects.

a. In which subject did you score the highest mark? How much was it?

b. In which subject did you score the lowest mark? How much was it?

c. How much was the difference between the highest and lowest marks?

d. What was your average mark/ result?

e. Do you get similar marks in different subjects? What was that score?

f. Arrange the marks in an increasing order? What is the middle score?

Once data have been collected (surveyed), they need to be organized so that the results can be understood and communicated.

You have to understand the mean and median as measures of center for different data sets. Here, the mean indicates the balance, the median indicates the middle of the data set and the mode shows the most frequently occurring values.

**Definition:**

1. **Mean:** The mean of the set of data is the value obtained by adding the values together and dividing by the number of values

   \[
   \text{Mean} = \frac{\text{Sum of values}}{\text{Number of values}}
   \]

2. **Mode:** The mode of a set of data is the value which occurs most frequently.

3. **Median:** For a data arranged in descending or ascending order, the median of the data is the *middle value* when the number of data is odd, and the average value of the two middle values when the number of data is even.

4. **Range:** The range of a set of data is the difference between the highest value and the lowest value:

   Range = highest value – lowest value
Example 7.6

Calculate the mean, median, mode and range of the following data.

12, 18, 9, 14, 8, 7

Solution:

a. Mean

Step 1: The sum of values is $12 + 18 + 9 + 14 + 8 + 7 = 68$

Step 2: Number of value is 6

Step 3: $\text{Mean} = \frac{\text{Sum of values}}{\text{Number of values}} = \frac{68}{6} \approx 11.3$

b. Mode is the value with the highest frequency. However, in the data each value appears only once. Therefore, no mode.

c. Median:

Arranging the data in an increasing order: 7, 8, 9, 12, 14, 18

Identifying the number of values: 6 (even)

For even number of values take the average of the two middle values:

$\text{Median} = \frac{9 + 12}{2} = 10.5$

d. The Range

Finding the largest and smallest values: 18 and 7

Therefore, Range = 18 - 7 = 11

Example 7.7

The following is recorded high room temperature ($^\circ$F) in one town. For the first week of January was 67$^\circ$F, 65$^\circ$F, 60$^\circ$F, 62$^\circ$F, 67$^\circ$F, 72$^\circ$F, and 71$^\circ$F. Find the mean, mode, median, and range of the given data.

Solution:

a. Mean

i. Sum of values = $67^\circ F + 65^\circ F + 60^\circ F + 62^\circ F + 67^\circ F + 72^\circ F + 71^\circ F = 464^\circ F$

ii. Number of value is 7

iii. $\text{Mean} = \frac{\text{Sum of values}}{\text{Number of values}} = \frac{464^\circ F}{7} \approx 66.3^\circ F$
b. **Mode:** The highest frequent value in the data is $67^\circ F$ (it appears two times)

Therefore, $67^\circ F$ is the mode of the data, and the data is uni-modal.

c. **Median:**

i. Arranged data: $60^\circ F, 62^\circ F, 65^\circ F, 67^\circ F, 67^\circ F, 71^\circ F, 72^\circ F$

ii. Number of values: 7 (odd number of values).

Therefore, the median is the 4th value in the sequence, which is $67^\circ F$.

d. **Range:**

i. Largest value = $72^\circ F$, the smallest value = $60^\circ F$

ii. Range = $72^\circ F - 60^\circ F = 12^\circ F$

**Example 7.8**

Ten teachers’ years of service in teaching profession are given. Compute the mean, median, mode and range of the data.

24, 19, 32, 20, 23, 28, 27, 27, 30, 24

**Solution:**

a. Mean $= \frac{24+19+32+20+23+28+27+27+30+24}{10} = \frac{254}{10} = 25.4$

Therefore, the average teaching service is about 25 years.

b. **Mode:**

24 19 32 20 23 28 27 27 30 24

Most teachers serve for 24 and 27 years in teaching. It is a bimodal.

c. **Median:**

19 20 23 24 24 27 27 28 30 32

Median $= \frac{24+27}{2} = 25.5$

d. **Range:**

24 19 27 20 23 28 27 32 30 24

Smallest Value

Largest Value

Thus, there is 13 years of service difference between the highest and lowest years of service.

In cases when a data has presented in frequencies table we compute the mean, mode, median and range as shown in the example below.

**Example 7.9**

The match-boxes are supposed to contain an average of 50 sticks. Merron decides to check the number of sticks. She takes a sample of 20 boxes, and counts the sticks in the boxes which are as shown in the frequency table.

<table>
<thead>
<tr>
<th>Number of match sticks (x)</th>
<th>48</th>
<th>49</th>
<th>50</th>
<th>52</th>
<th>53</th>
<th>54</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequencies (f)</td>
<td>1</td>
<td>5</td>
<td>7</td>
<td>5</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

Compute the mean, mode, median and range of the data.

**Solution:**

**Mean:** First multiply the number with their frequency, and add up the frequencies \( f \) and the product of the number with the frequencies \( xf \).

<table>
<thead>
<tr>
<th>Number of matches (f)</th>
<th>Frequency (xf)</th>
<th>xf</th>
</tr>
</thead>
<tbody>
<tr>
<td>48</td>
<td>1</td>
<td>48×1 = 48</td>
</tr>
<tr>
<td>49</td>
<td>5</td>
<td>49×5 = 245</td>
</tr>
<tr>
<td>50</td>
<td>7</td>
<td>50×7 = 350</td>
</tr>
<tr>
<td>52</td>
<td>5</td>
<td>52×5 = 260</td>
</tr>
<tr>
<td>53</td>
<td>1</td>
<td>53×1 = 53</td>
</tr>
<tr>
<td>54</td>
<td>1</td>
<td>54×1 = 54</td>
</tr>
<tr>
<td></td>
<td>20</td>
<td>1010</td>
</tr>
</tbody>
</table>

Divide sum of the product by the sum of the frequencies

\[
Mean = \frac{\text{Sum of values}}{\text{Number of values}} = \frac{1010}{20} = 50.5
\]
Mode: The highest frequent value is 7. The value corresponding with 7 is 50.

<table>
<thead>
<tr>
<th>Number of matches sticks</th>
<th>48</th>
<th>49</th>
<th>50</th>
<th>51</th>
<th>52</th>
<th>53</th>
<th>54</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequencies</td>
<td>1</td>
<td>5</td>
<td>7</td>
<td>0</td>
<td>5</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

Therefore, 50 is the mode and the data is uni-modal.

Median: To find the median, identify the middle number in an ordered data set.

<table>
<thead>
<tr>
<th>Number of matches, x</th>
<th>48</th>
<th>49</th>
<th>50</th>
<th>52</th>
<th>53</th>
<th>54</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency, ( f )</td>
<td>1</td>
<td>5</td>
<td>7</td>
<td>5</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

Total

The median for \( n \) items is \( \left( \frac{n + 1}{2} \right)^{th} \) term, where \( n \) is the number of items.

\[
\left( \frac{n + 1}{2} \right)^{th} = \left( \frac{20 + 1}{2} \right)^{th} = 10.5
\]

Thus, 10.5 is between 10 and 11, hence the median is the average of the 10\(^{th}\) and the 11\(^{th}\) value in the sequence. Both the 10\(^{th}\) and 11\(^{th}\) values are 50.

Therefore, the median = \( \frac{50 + 50}{2} = 50 \).

Range: The highest value is 54 and the lowest is 48.

Range = 54 - 48 = 6

**Exercise 7.5**

1. Find the mean, mode, median and range of 9 mathematics students first semester exam results and express what the mean, median, mode, and range of the data are meant about the students’ result.

78, 83, 84, 86, 87, 90, 92, 92, 94
2. The following are number of castle palaces found in Amhara Region. Find the mean, mode, median, and range of the numbers.

<table>
<thead>
<tr>
<th>Zone / Place</th>
<th>Number of Palace</th>
</tr>
</thead>
<tbody>
<tr>
<td>West Gojjam</td>
<td>6</td>
</tr>
<tr>
<td>Dessie</td>
<td>2</td>
</tr>
<tr>
<td>Centeral Gondar</td>
<td>4</td>
</tr>
<tr>
<td>South Wollo</td>
<td>10</td>
</tr>
<tr>
<td>North Wollo</td>
<td>8</td>
</tr>
<tr>
<td>Gondar town</td>
<td>8</td>
</tr>
<tr>
<td>South Gondar</td>
<td>5</td>
</tr>
<tr>
<td>Bahir Dar</td>
<td>1</td>
</tr>
</tbody>
</table>

3. Find the mean, mode(s), median and range of the data given in the table below.

<table>
<thead>
<tr>
<th>Age of students</th>
<th>Tally marks</th>
<th>Frequency (Number of students)</th>
</tr>
</thead>
<tbody>
<tr>
<td>13</td>
<td>13</td>
<td>13</td>
</tr>
<tr>
<td>14</td>
<td>17</td>
<td>17</td>
</tr>
<tr>
<td>15</td>
<td>16</td>
<td>16</td>
</tr>
<tr>
<td>16</td>
<td>14</td>
<td>14</td>
</tr>
<tr>
<td>17</td>
<td>6</td>
<td>6</td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td>66</td>
</tr>
</tbody>
</table>

4. The mean of four numbers is 94, and the mean of another nine different numbers is 17. What is the mean of all thirteen numbers?

5. Find the value of x so that the mean of the given data is 8
   14, 6, 2x, 8, 10, 4.

6. Use the information given to find the value of x in each of the following data.
   a. 2, x, 5, 7, 1, 3: the median is \( \frac{7}{2} \).
   b. 4, 7, 2, x, 2, 9, 6: the median is 5

7. Find the range of these sets of data: -2,-9,-1,-2000,-6000.
8. The range for the nine numbers shown below is 40. Find the two possible values of the missing number.

<table>
<thead>
<tr>
<th>13</th>
<th>5</th>
<th>27</th>
</tr>
</thead>
<tbody>
<tr>
<td>11</td>
<td>19</td>
<td>?</td>
</tr>
<tr>
<td>42</td>
<td>11</td>
<td>33</td>
</tr>
</tbody>
</table>

9. Calculate the mode(s) for each of the following data.
   a. 1100, 966, 688, 499, 366, 1278, 1000, 699, 566
   b. 1106, 1207, 1138, 1166, 1188, 1196, 1278, 1179, 1186, 1186, 1138

Problem Solving

The following table shows how much percent of public elementary schools have an Internet access. The data where gather from 2008 to 2013E.C.

<table>
<thead>
<tr>
<th>Year</th>
<th>2006</th>
<th>2007</th>
<th>2008</th>
<th>2009</th>
<th>2010</th>
<th>2011</th>
<th>2012</th>
<th>2013</th>
</tr>
</thead>
<tbody>
<tr>
<td>Access in Percent(%)</td>
<td>30</td>
<td>46</td>
<td>61</td>
<td>75</td>
<td>88</td>
<td>95</td>
<td>98</td>
<td>99</td>
</tr>
</tbody>
</table>

   a. Draw a line graph with the years from 2006 to 2013E.C.
   b. During which two-year period did the elementary schools with Internet access approximately doubled?

Activity 7.7

How can we compute the mean, median and mode using spreadsheet (excel sheet)

Example 7.10

The following is a list of the top ten salaries of a company in a recent year. Make a spreadsheet for the data.

<table>
<thead>
<tr>
<th>Top Ten Annual Salaries in Birr</th>
</tr>
</thead>
<tbody>
<tr>
<td>851,198</td>
</tr>
<tr>
<td>485,333</td>
</tr>
</tbody>
</table>
UNIT 7: DATA HANDLING

Compute the mean, median and mode using spread sheet (excel sheet).

Solution:

To calculate the mean of a dataset, use the AVERAGE function and set the input as the area of the array containing the data. For our example, the data for Annual salaries covers cells A2 to A11:

=AVERAGE(A2:A11)

In a similar fashion, the MEDIAN() function calculates the median of the array. For our example, the data for the amount of items ordered covers cells A2 to A11:

=MEDIAN(A2:A11)

Finally, the MODE() function calculates the mode of the array. For our example, the data for the amount of items ordered covers cells A2 to A11:

=MODE(A2:A11)

Exercise 7.6

1. Two companies have ten employs each with Annual salaries in Birr as shown in the table below.

<table>
<thead>
<tr>
<th>Annual Salaries in company A</th>
<th>Annual Salaries in company B</th>
</tr>
</thead>
<tbody>
<tr>
<td>455,425</td>
<td>750,000</td>
</tr>
<tr>
<td>500,000</td>
<td>249,411</td>
</tr>
<tr>
<td>962,703</td>
<td>550,000</td>
</tr>
<tr>
<td>800,000</td>
<td>843,666</td>
</tr>
<tr>
<td>783,600</td>
<td>600,000</td>
</tr>
</tbody>
</table>

   a. Use spreadsheets to find the mean, median and mode of the salaries in each company.

   b. Compare the highest salary for the three positions.

   c. Compare the mean and median of the salaries in these companies.

2. The following table shows ten merchants amount of potato they purchases in kilograms, the selling prices and profits each gained.

   Use Spread sheet to calculate the mean profit, the median of the amount of items ordered/ purchased, and the mode of the amount of items ordered.
<table>
<thead>
<tr>
<th>Merchants</th>
<th>Amount of Potato (in k/g) ordered</th>
<th>Principal</th>
<th>Profit</th>
<th>Total price</th>
</tr>
</thead>
<tbody>
<tr>
<td>Almaz</td>
<td>5</td>
<td>50</td>
<td>10</td>
<td>60</td>
</tr>
<tr>
<td>Hailu</td>
<td>23</td>
<td>230</td>
<td>46</td>
<td>276</td>
</tr>
<tr>
<td>Jemal</td>
<td>16</td>
<td>160</td>
<td>32</td>
<td>192</td>
</tr>
<tr>
<td>Kidist</td>
<td>12</td>
<td>120</td>
<td>24</td>
<td>144</td>
</tr>
<tr>
<td>Ansha</td>
<td>7</td>
<td>70</td>
<td>14</td>
<td>84</td>
</tr>
<tr>
<td>Tirru</td>
<td>21</td>
<td>210</td>
<td>42</td>
<td>252</td>
</tr>
<tr>
<td>Kebede</td>
<td>10</td>
<td>100</td>
<td>20</td>
<td>120</td>
</tr>
<tr>
<td>Molu</td>
<td>25</td>
<td>250</td>
<td>50</td>
<td>300</td>
</tr>
<tr>
<td>Rukeya</td>
<td>3</td>
<td>30</td>
<td>6</td>
<td>36</td>
</tr>
<tr>
<td>Abebe</td>
<td>9</td>
<td>90</td>
<td>18</td>
<td>108</td>
</tr>
</tbody>
</table>

**Unit summary**

- Line graph and pie chart are some ways of representing data.

- A good line graph or pie chart should contain the following points:
  - Have a title.
  - Be well proportioned.
  - Have scales clearly marked and labeled.
  - Show source of the facts that it represents.

- The mean of data is the value obtained by adding the values together and dividing by the number of values.

  \[ \text{Mean} = \frac{\text{Sum of values}}{\text{Number of values}} \]

- The mode of a set of data is the value which occurs most frequently.

- The median of the data with n number of items arranged in order of size is:
  - The middle single item if n is odd
  - The average of the two middle items if n is even.
UNIT 7: DATA HANDLING

- Range = highest value – lowest value.
- A data that has a unique mode is called uni-modal.
- A set of data which has two modes is called bi-modal.
- A set of data that has three modes is called Tri-modal.
- If each value occurs only once, there is no mode at all.

Review Exercises

1. Make frequency distribution table with a tally mark for each of the following raw data
   a. 2, 3, 4, 3, 4, 3, 8, 7, 2
   b. 31, 24, 26, 18, 23, 31, 18
   c. 89, 76, 93, 100, 72, 86, 74
   d. 54,000, 49,000, 112,000, 89,000, 76,000, 65,000

2. Pie chart is constructed to show the data gathered after a survey of 200 people: Where would you like to go on holiday?
   a. How many people wanted to go to visit Bahir Dar on holidays?
   b. Calculate the mean of the data

![Pie Chart]

### Visitors Preference

<table>
<thead>
<tr>
<th>Key</th>
<th>Dessie</th>
<th>Bahir Dar</th>
<th>Lalibela</th>
<th>Gondar</th>
</tr>
</thead>
<tbody>
<tr>
<td>96°</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>80°</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>84°</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>100°</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

3. Compute the mean, median, mode, and Range for the following Group A and Group B data. What differences and similarities have you observed?

<table>
<thead>
<tr>
<th>Group A</th>
<th>13</th>
<th>14</th>
<th>15</th>
<th>19</th>
<th>20</th>
<th>20</th>
<th>28</th>
<th>29</th>
<th>30</th>
<th>32</th>
<th>33</th>
</tr>
</thead>
<tbody>
<tr>
<td>Group B</td>
<td>1</td>
<td>4</td>
<td>15</td>
<td>16</td>
<td>17</td>
<td>20</td>
<td>20</td>
<td>21</td>
<td>30</td>
<td>31</td>
<td>78</td>
</tr>
</tbody>
</table>

4. Group A and Group B were asked to vote on one of the four clubs established in their school. The data is represented in these pie charts.
Which of these statements are true?

a. Fewer students voted for Mathematics Club in Group B than Group A

b. More students voted for ICT club in Group A than Group B

c. A higher proportion of students voted for Environmental Science club in Group B than Group A

d. Number of students at group A and B might not be equal.

5. The following chart shows the number of families in a certain village with the number of children each family has.

<table>
<thead>
<tr>
<th>Number of Children</th>
<th>Tally marks</th>
<th>Frequency (Number of students)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>/ / / /</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>/ / / / / /</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>/ / /</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>/ /</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>/</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>/</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Total</td>
<td></td>
</tr>
</tbody>
</table>

Complete the last column and compute the mean, mode, median and range.

6. Find the mean, mode(s), median and range for each collection of data

a. 2, 3, 4, 3, 4, 3, 8, 7, 2

b. 89, 76, 93, 100, 72, 86, 74

c. 31, 24, 26, 18, 23, 31, 18

d. 54,000, 49,000, 112,000, 89,000, 76,000, 65,000
7. Determine the mean, median, and mode of the following salaries for people in a small company.

<table>
<thead>
<tr>
<th>Position</th>
<th>Salary (Birr)</th>
</tr>
</thead>
<tbody>
<tr>
<td>One president</td>
<td>25,000</td>
</tr>
<tr>
<td>One vice-president</td>
<td>20,000</td>
</tr>
<tr>
<td>One salesperson</td>
<td>16,000</td>
</tr>
<tr>
<td>One supervisor</td>
<td>14,000</td>
</tr>
<tr>
<td>One machine operator</td>
<td>14,000</td>
</tr>
<tr>
<td>Five mill workers (each earning)</td>
<td>8,000</td>
</tr>
<tr>
<td>Six contract workers (each earning)</td>
<td>5,000</td>
</tr>
</tbody>
</table>

8. The following table shows HIV aids infected people in Ethiopia in the specified years. Determine the four years average (mean) HIV infection of males, females and total.

<table>
<thead>
<tr>
<th>Variables</th>
<th>Year</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>2015</td>
<td>2016</td>
</tr>
<tr>
<td>HIV positive (Male)</td>
<td>6,800</td>
<td>5,901</td>
</tr>
<tr>
<td>HIV positive (Female)</td>
<td>11,142</td>
<td>6,616</td>
</tr>
<tr>
<td>HIV positive (Total)</td>
<td>17,942</td>
<td>12,517</td>
</tr>
</tbody>
</table>

Source: from the internet